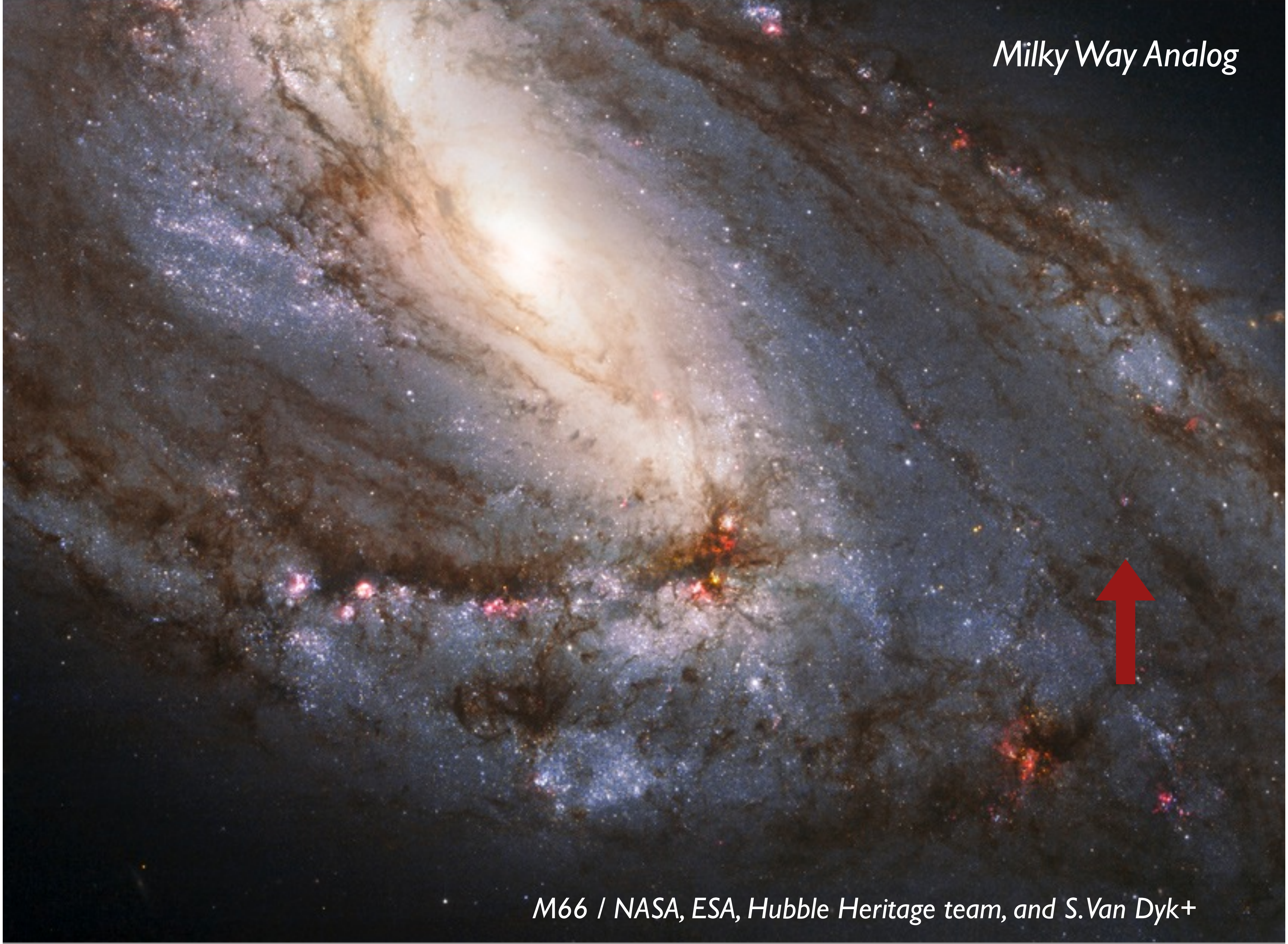
General Relativistic Magnetohydrodynamics

Charles F. Gammie
Physics & Astronomy, University of Illinois
Oxford ('15-'16; sabbatical)

Les Houches, May 2016



Milky Way Analog

M66 / NASA, ESA, Hubble Heritage team, and S. Van Dyk+

Urbana, Illinois

University of Illinois at Urbana-Champaign

Blue Waters Supercomputer
University of Illinois

Relativistic MHD

- Lecture 1: Astrophysical Motivation
- Lecture 2: Basic Equations, GRMHD Boot Camp
- Lecture 3: Numerical Methods
- Lecture 4: Radiative Transport and Analysis

Lecture 1: Astrophysical Motivation

- What is relativistic (magneto)hydrodynamics?
- What are the (astro)physical applications?
- Accreting black holes
- Open questions

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Application Summary

black holes:
- tidal disruption events
- high, low accretion rate AGN
- x-ray binaries
- long-soft GRBs
- BH-BH and BH-NS mergers,
  - LIGO source progenitors

neutron stars:
- pulsar magnetospheres
- core-collapse supernovae
- short-hard GRBs
- NS-NS mergers
  - LIGO source progenitors

jets:
- gamma-ray bursts
- extragalactic radio jets
- galactic microquasars

planetary magnetospheres

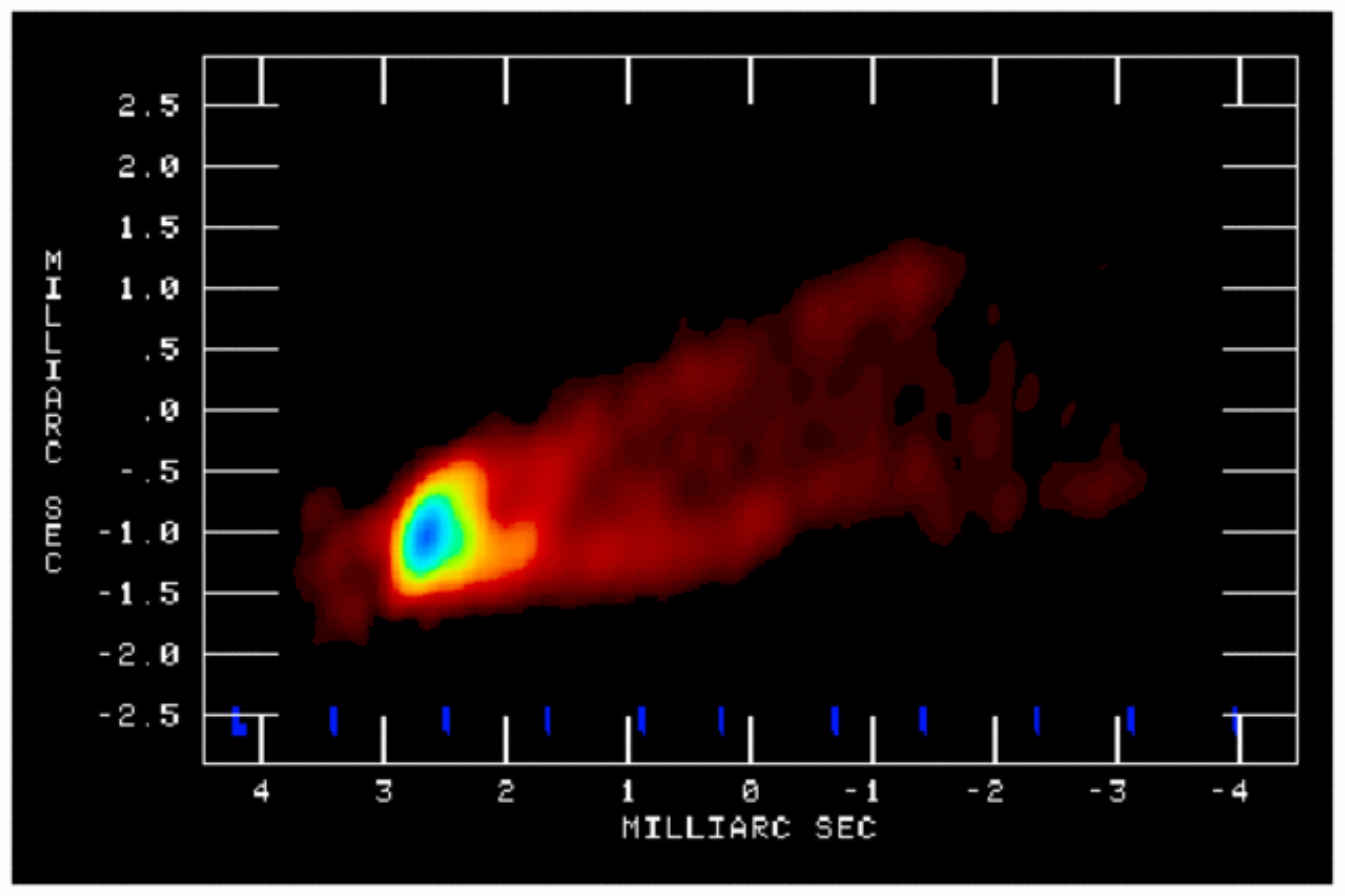
relativistic heavy ion collisions

electron-fluid dynamics

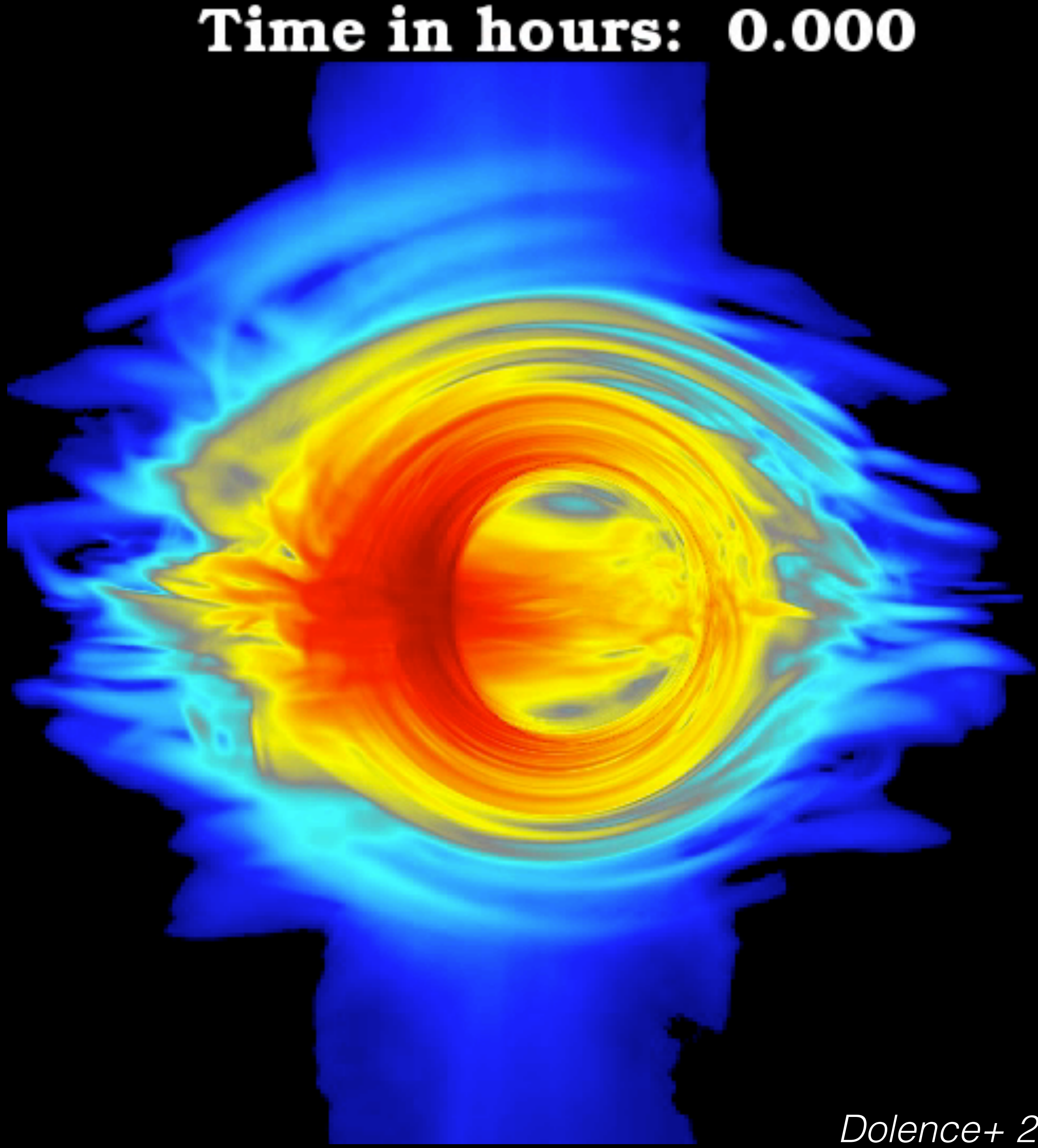
Tidal Disruption Event

Shiokawa+ 2015

M87 Jet, 43 GHz, VLBA



Walker+, NRAO



log 1mm
surface
brightness

Dolence+ 2009

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)

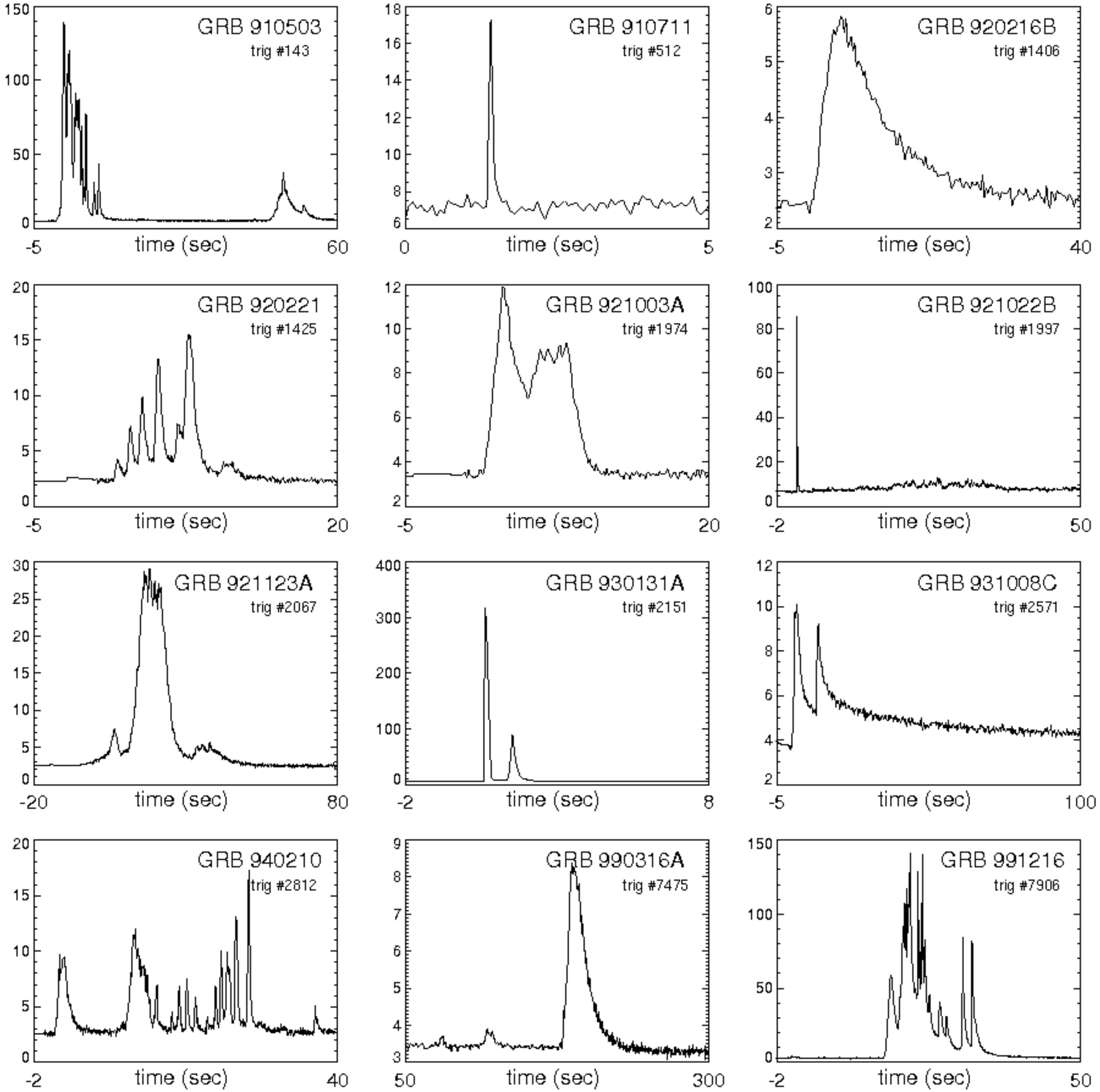
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals.

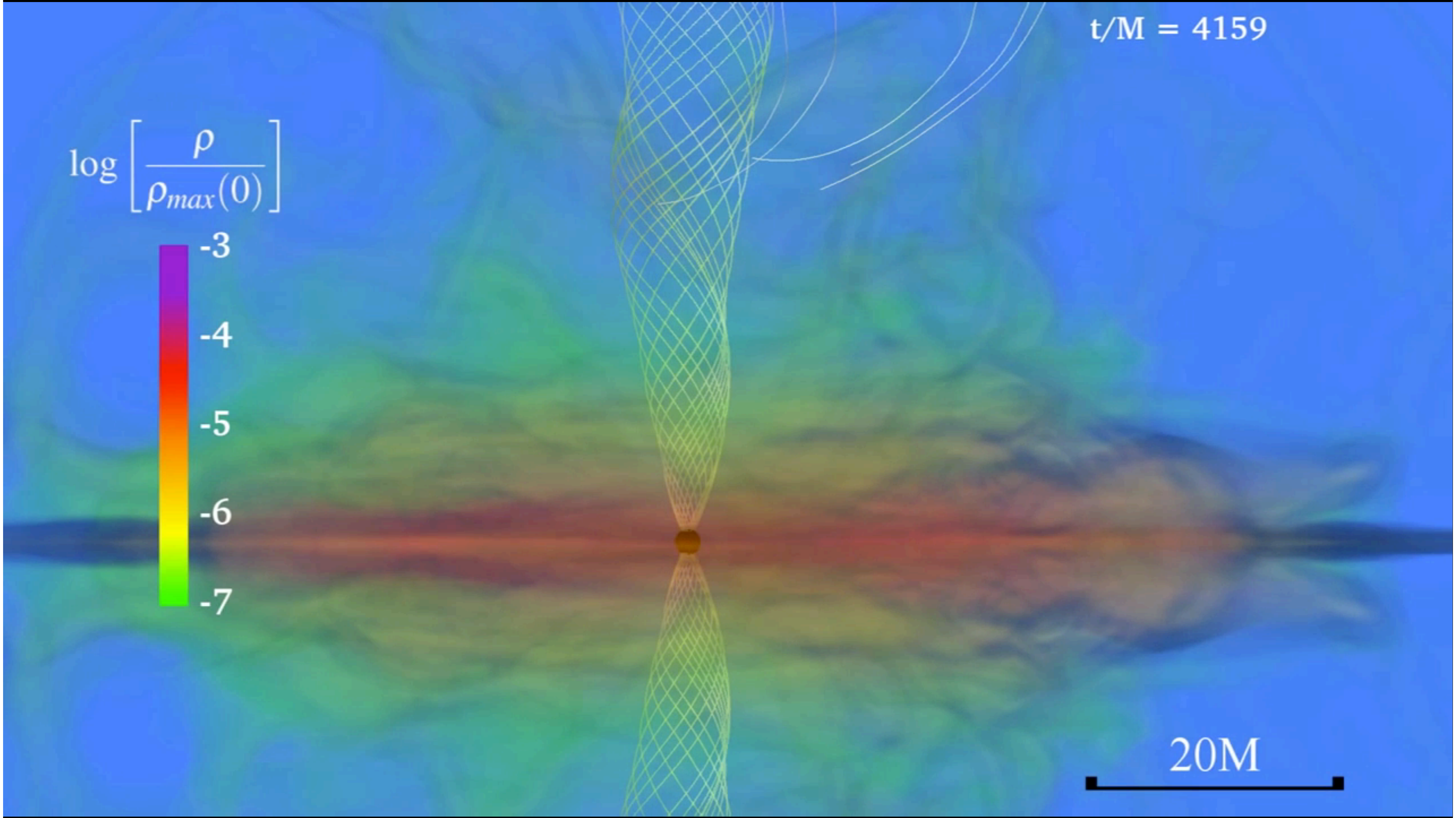
Fermi GBM Observations of LIGO Gravitational Wave event GW150914

V. Connaughton[*,1], E. Burns[2], A. Goldstein[+,3], M. S. Briggs[4], B.-B. Zhang[5] C. M. Hui[3], P. Jenke[5], J. Racusin[6], C. A. Wilson-Hodge[3], P. N. Bhat[5], E. Bissaldi[7], W. Cleveland[1], G. Fitzpatrick[5], M. M. Giles[8], M. H. Gibby[8], J. Greiner[9], A. von Kienlin[9], R. M. Kippen[10], S. McBreen[11], B. Mailyan[5], C. A. Meegan[5], W. S. Paciesas[1], R. D. Preece[4], O. Roberts[10], L. Sparke[12], M. Stanbro[2], K. Toelge[9], P. Veres[5], H.-F. Yu[9,13]

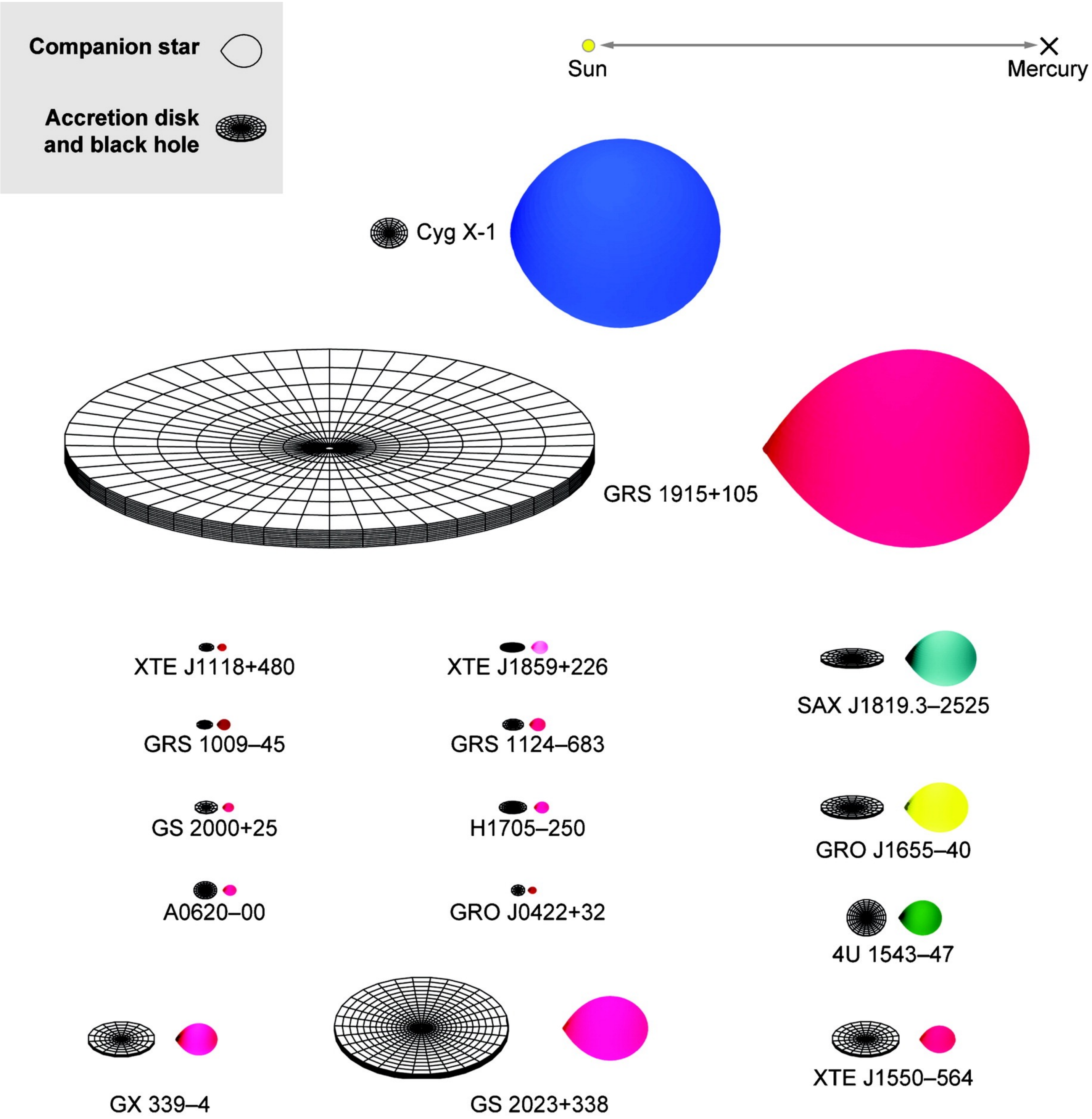
and other authors


GRB 910503
trig #143
0
50
100
150
-5
60
time (sec)
GRB 910711
trig #512
6
8
10
12
14
16
18
0
5
time (sec)
GRB 920216B
trig #1406
2
3
4
5
6
-5
40
time (sec)
GRB 920221
trig #1425
0
5
10
15
20
-5
20
time (sec)
GRB 921003A
trig #1974
2
4
6
8
10
12
-5
20
time (sec)
GRB 921022B
trig #1997
0
20
40
60
80
100
-2
50
time (sec)
GRB 921123A
trig #2067
0
5
10
15
20
25
30
-20
80
time (sec)
GRB 930131A
trig #2151
0
100
200
300
400
-2
8
time (sec)
GRB 931008C
trig #2571
2
4
6
8
10
12
-5
100
time (sec)
GRB 940210
trig #2812
0
5
10
15
20
-2
40
time (sec)
GRB 990316A
trig #7475
3
4
5
6
7
8
9
50
300
time (sec)
GRB 991216
trig #7906
0
50
100
150
-2
50
time (sec)

NS-NS mergers / short-hard GRB?



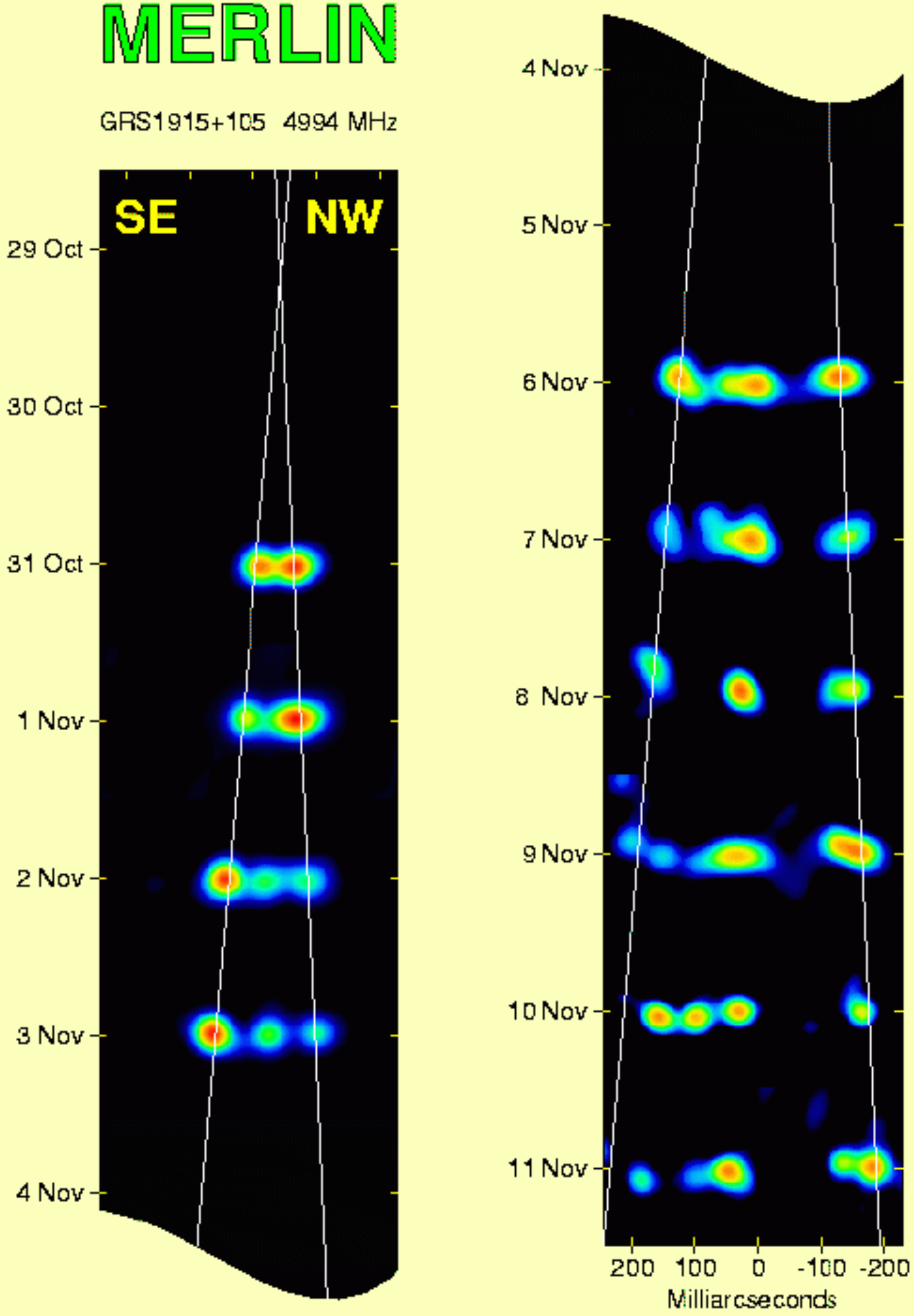
Shapiro, Illinois Relativity Group



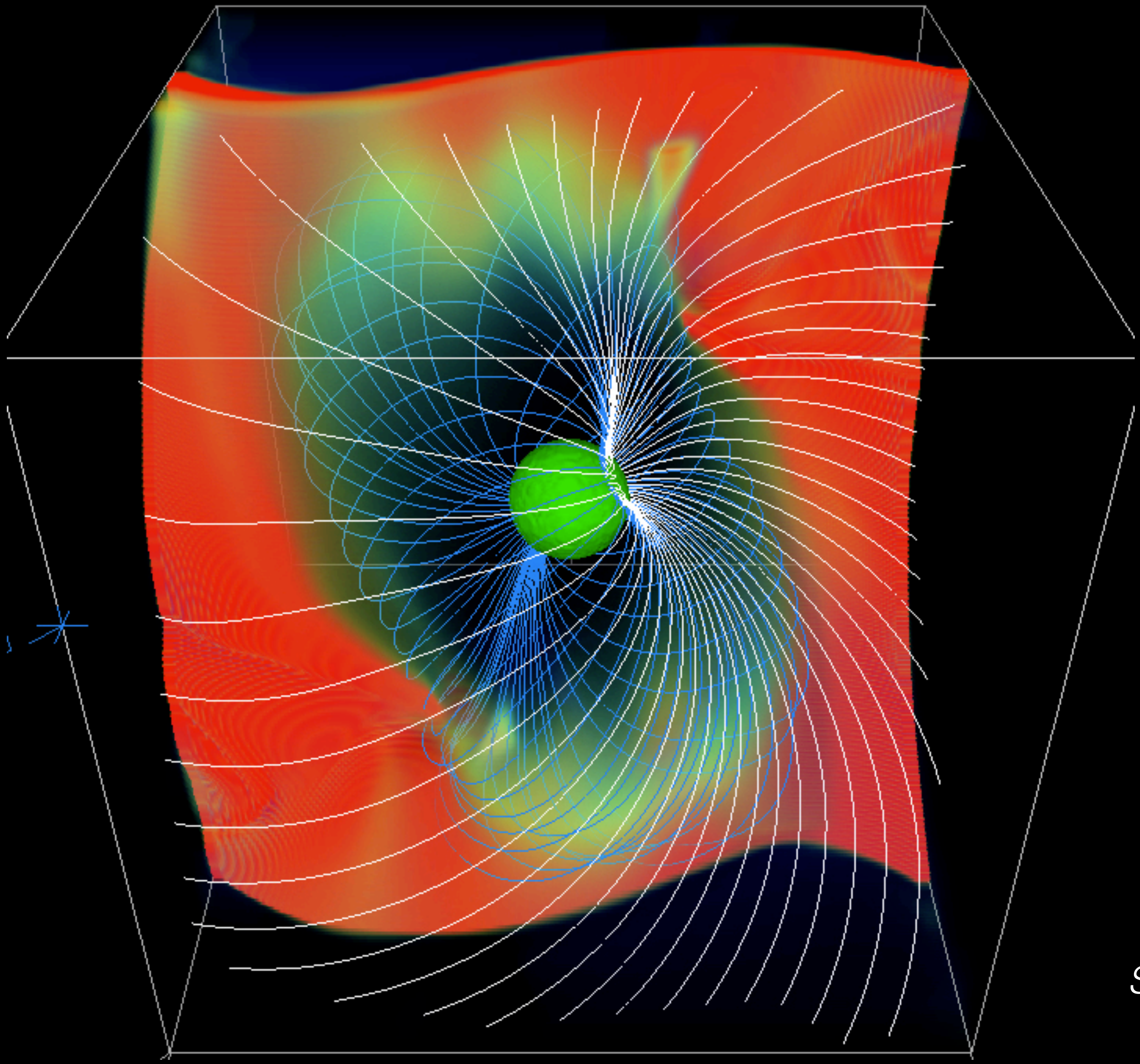
Remillard RA, McClintock JE. 2006.
Annu. Rev. Astron. Astrophys. 44:49–92

MERLIN

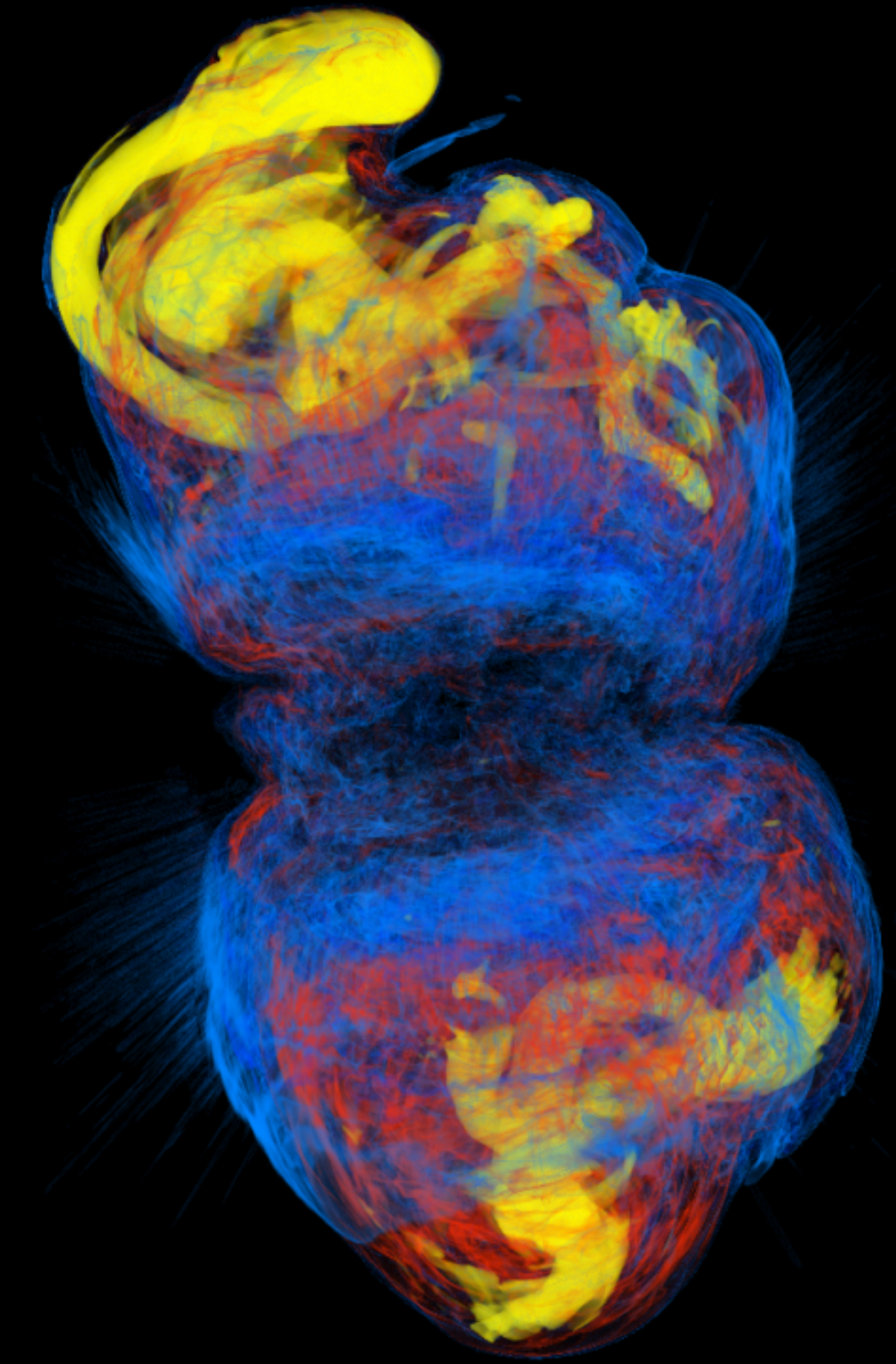
GRS1915+105 4994 MHz



Pulsar Magnetosphere - Force-Free MHD



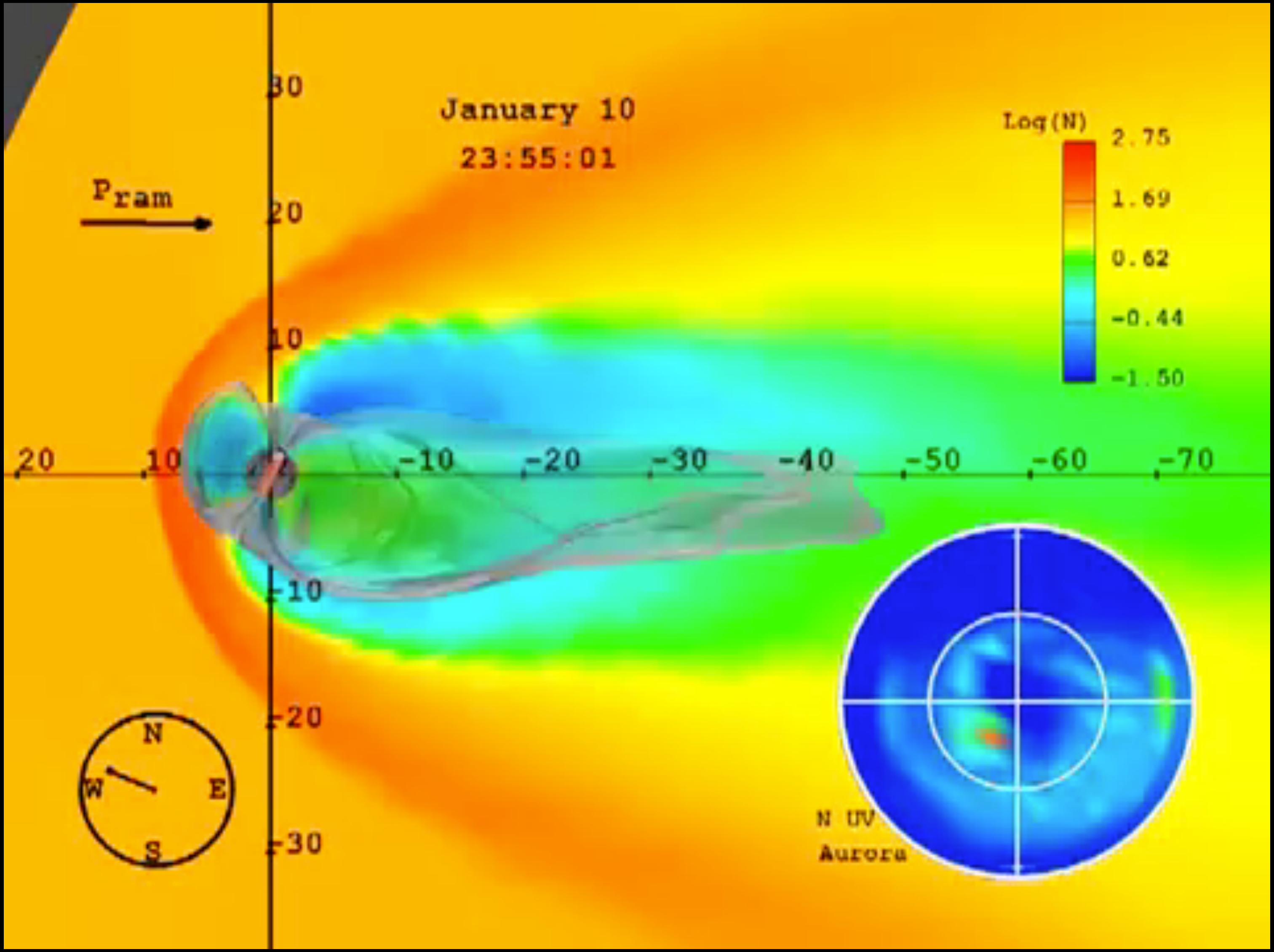
Spitkovsky



core-collapse
supernova

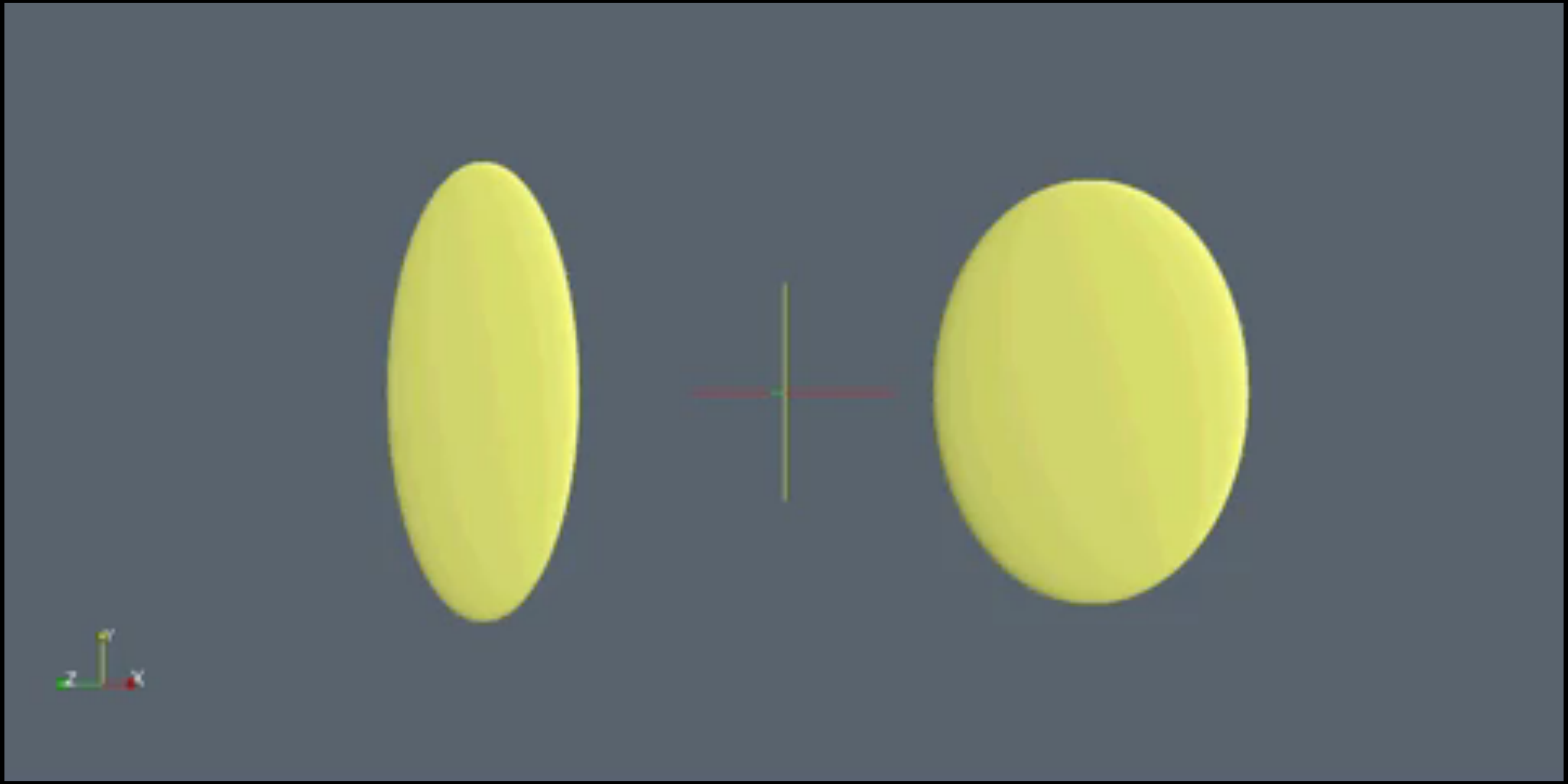
Mösta+ 2014

Earth's Magnetosphere



NASA

Relativistic Heavy Ion Collision



T. Hirano

ELECTRON TRANSPORT

Evidence for hydrodynamic electron flow in $PdCoO_2$

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3]
Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4]*

Electron transport is conventionally determined by the momentum-relaxing scattering of electrons by the host solid and its excitations. Hydrodynamic fluid flow through channels, in contrast, is determined partly by the viscosity of the fluid, which is governed by momentum-conserving internal collisions. A long-standing question in the physics of solids has been whether the viscosity of the electron fluid plays an observable role in determining the resistance. We report experimental evidence that the resistance of restricted channels of the ultrapure two-dimensional metal palladium cobaltate ($PdCoO_2$) has a large viscous contribution. Comparison with theory allows an estimate of the electronic viscosity in the range between 6×10^{-3} kg m^{-1} s^{-1} and 3×10^{-4} kg m^{-1} s^{-1}, versus 1×10^{-3} kg m^{-1} s^{-1} for water at room temperature.

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Modes of Accretion

Low accretion rate

$\dot{M} \ll \dot{M}_{Edd} \equiv L_{Edd} / (\epsilon c^2)$

Radiatively Inefficient Accretion Flow (RIAF)

optically thin, geometrically thick

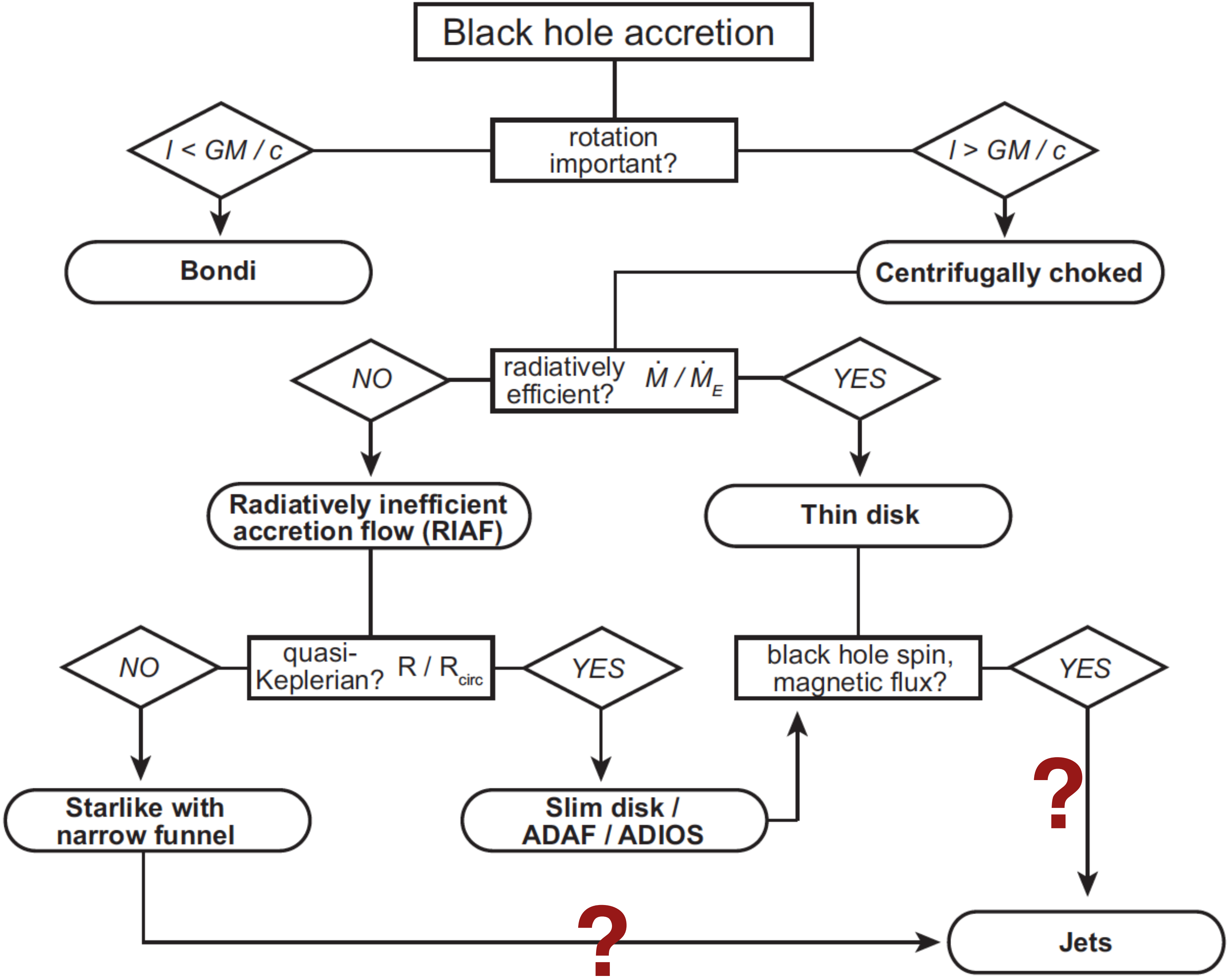
High accretion rate

$\dot{M} \gtrsim 10^{-2} \dot{M}_{Edd}$

classical disk (Shakura-Sunyaev)

optically thick, geometrically thin

Begelman 2014



Black hole accretion

rotation important?

$l < GM/c$

Bondi

$l > GM/c$

Centrifugally choked

radiatively efficient? $\dot{M}/\dot{M}_E$

NO

Radiatively inefficient accretion flow (RIAF)

YES

Thin disk

quasi-Keplerian? R/R_{circ}

NO

Starlike with narrow funnel

YES

Slim disk / ADAF / ADIOS

black hole spin, magnetic flux?

YES

?

?

Jets

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Modes of Accretion

Low accretion rate   Galactic center: Sgr A*

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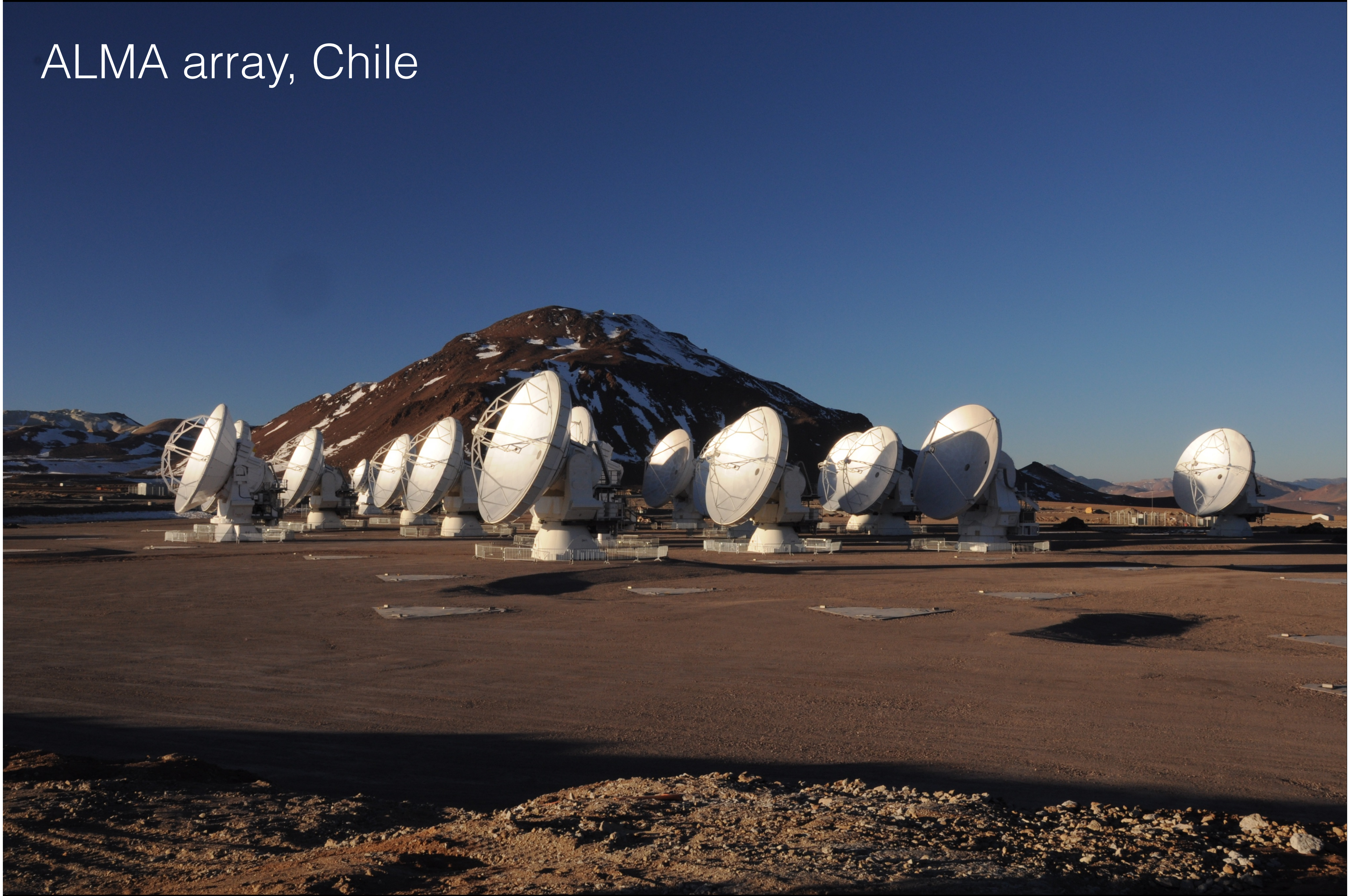
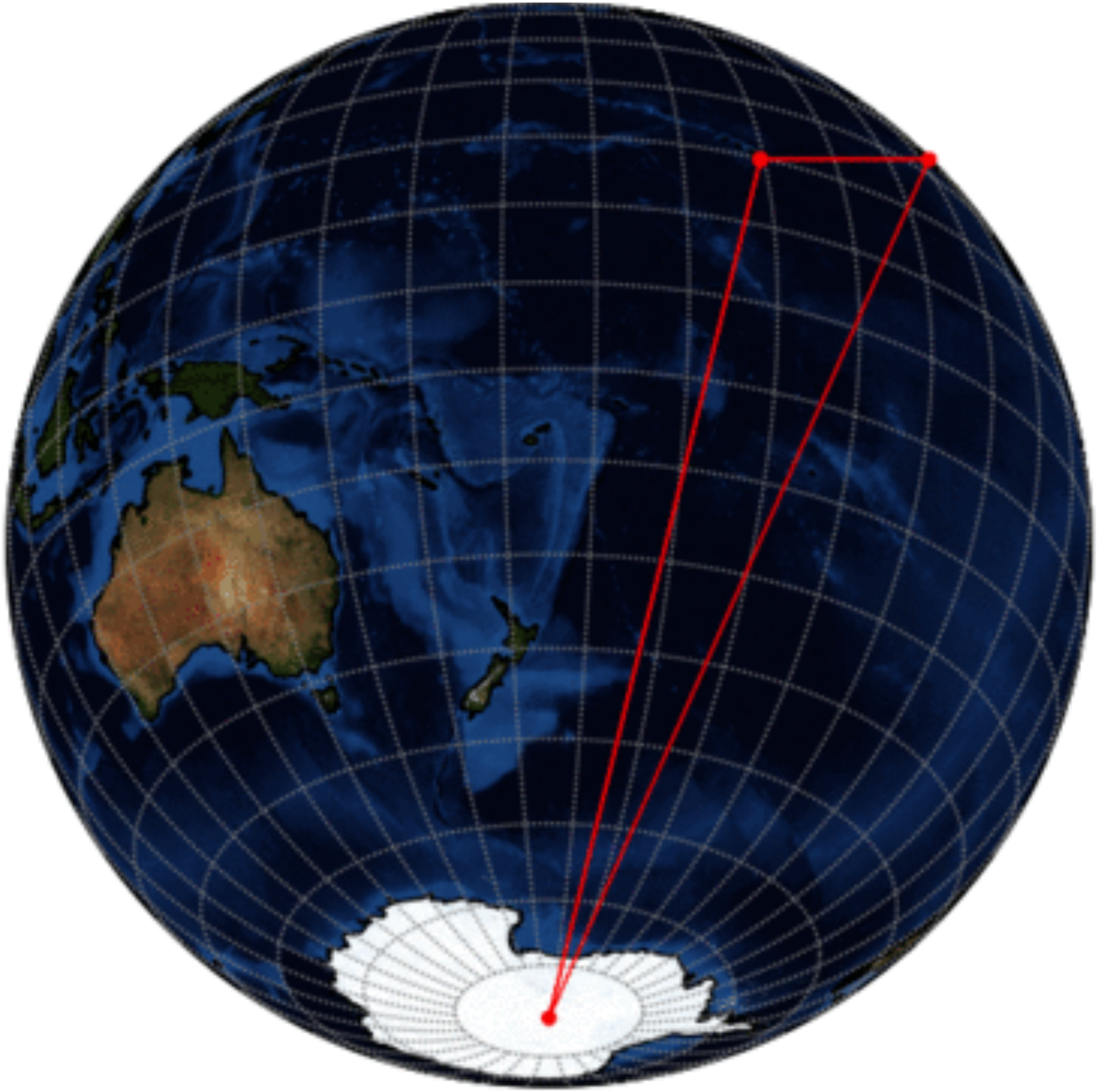
European Southern Observatory

ALMA array, Chile



South Pole Telescope

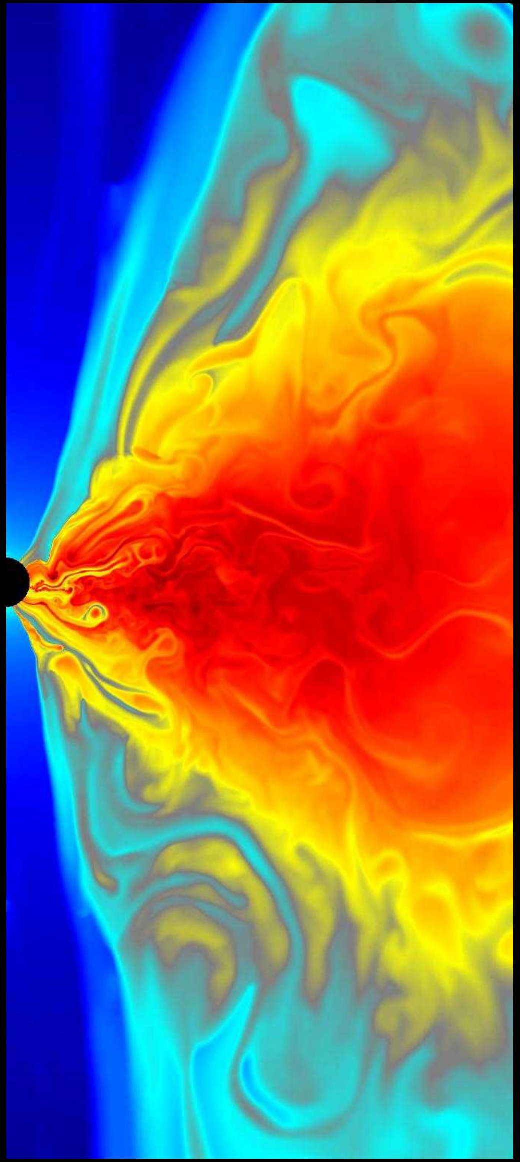
Event Horizon Telescope: global submm VLBI network

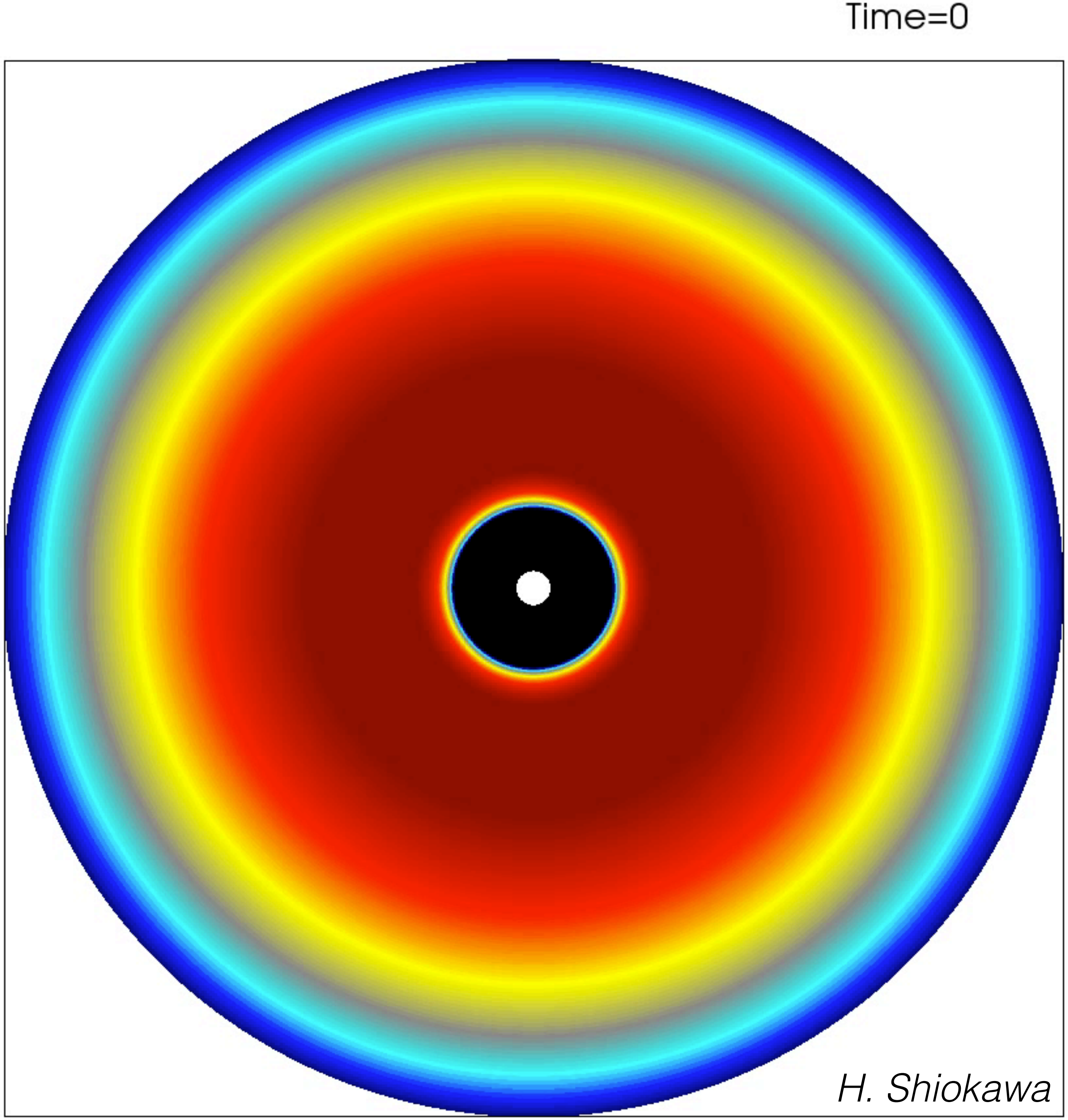


courtesy L. Vertatschitsch

Dynamical Model

- ideal MHD (fluid)
- fully relativistic
- Kerr metric (includes spin *a*)
- nonradiative
- `harm` code





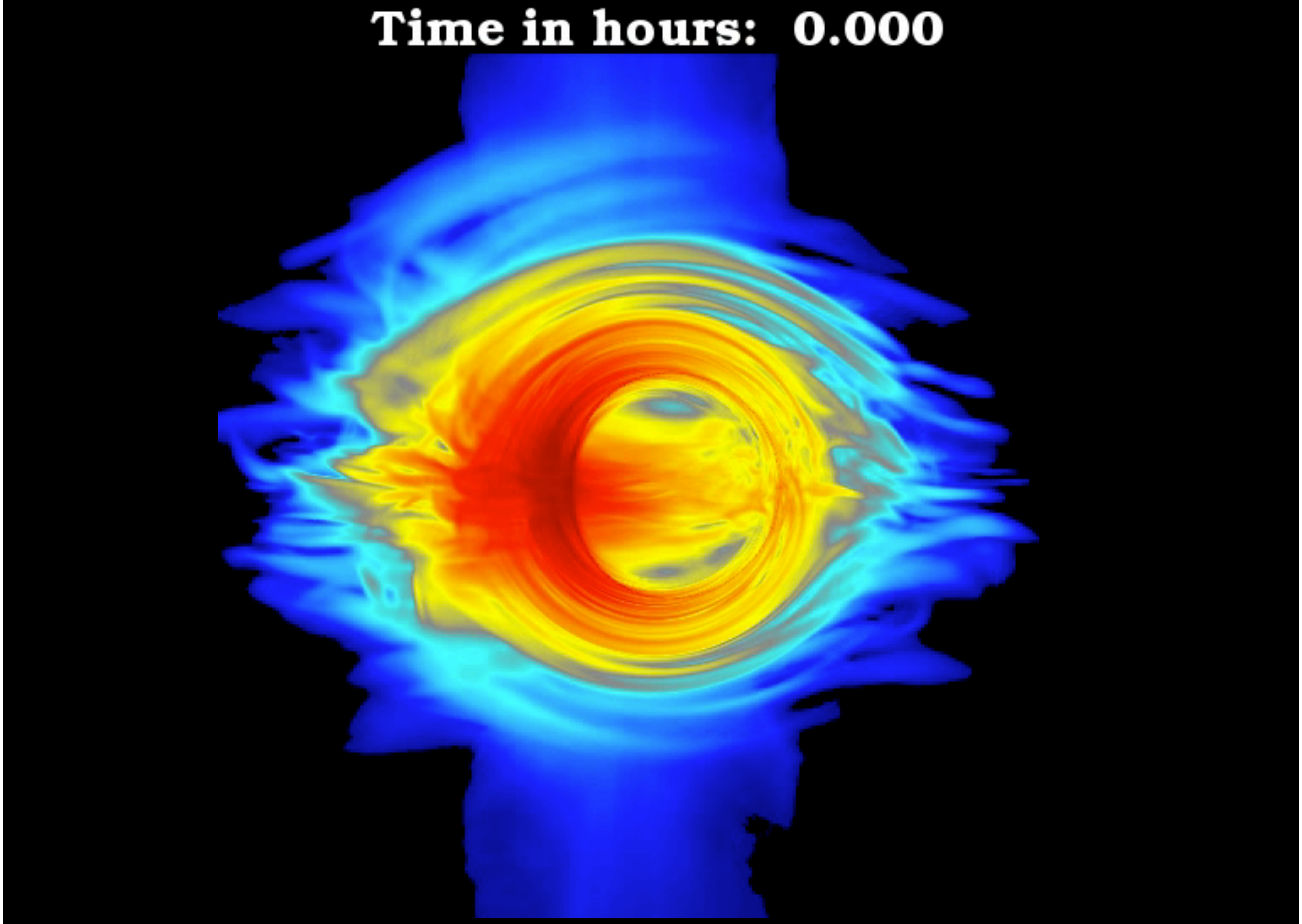
Noble's
`harm3d`

Color:
Rest-mass
density

Equatorial
plane

Time unit:
20s

a/M ~ 0.93



1.3mm surface brightness / J Dolence

`harm` code

go to

https://github.com/AFD-Illinois/iharm2d_v3

download zip file, `unzip`

```
cd iharm2d_v3-master
```

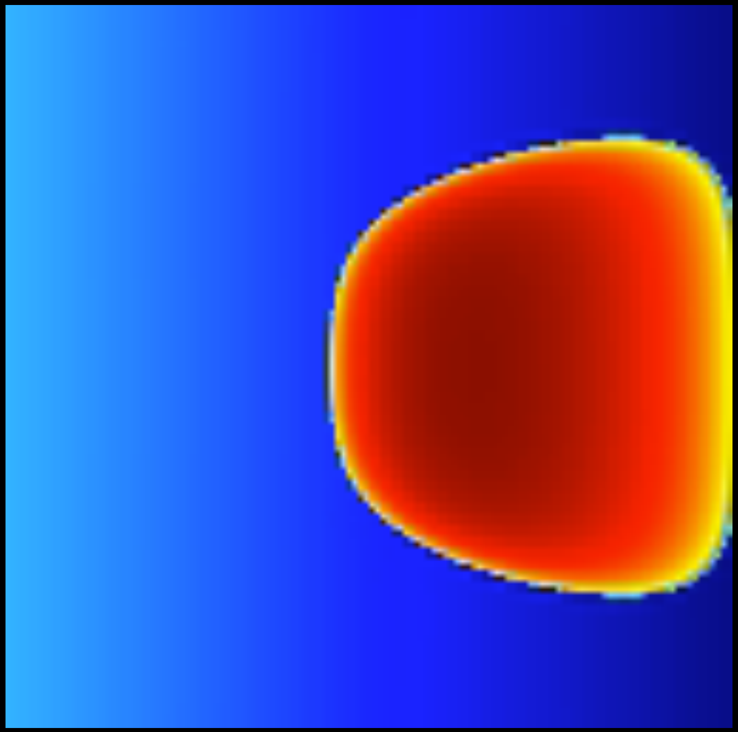
`README` file describes typical workflow

```
cd prob:kerr_torus
```

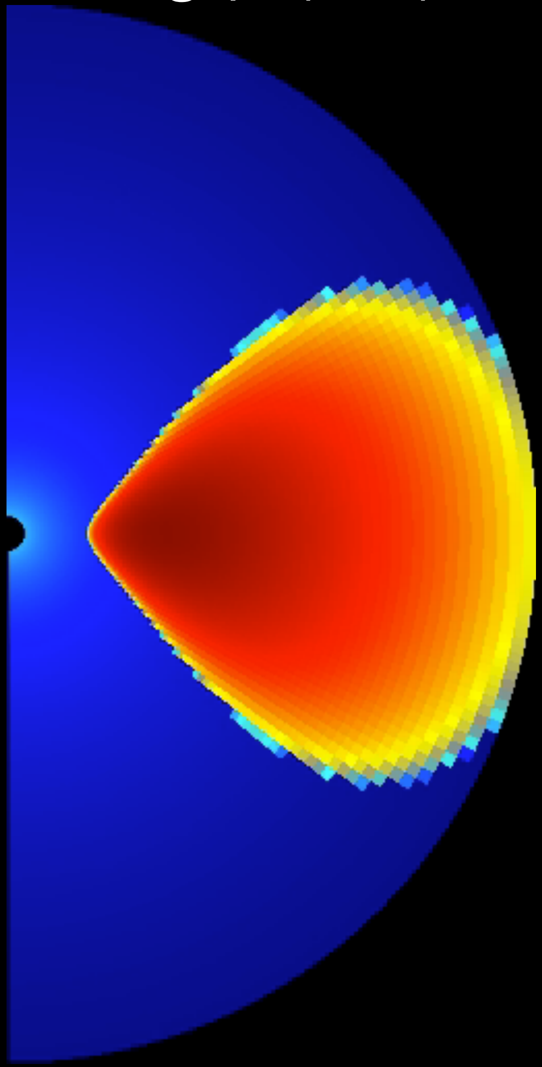
```
make
```

run: `./harm`

log ρ (x_1, x_2)



log ρ (r, θ)



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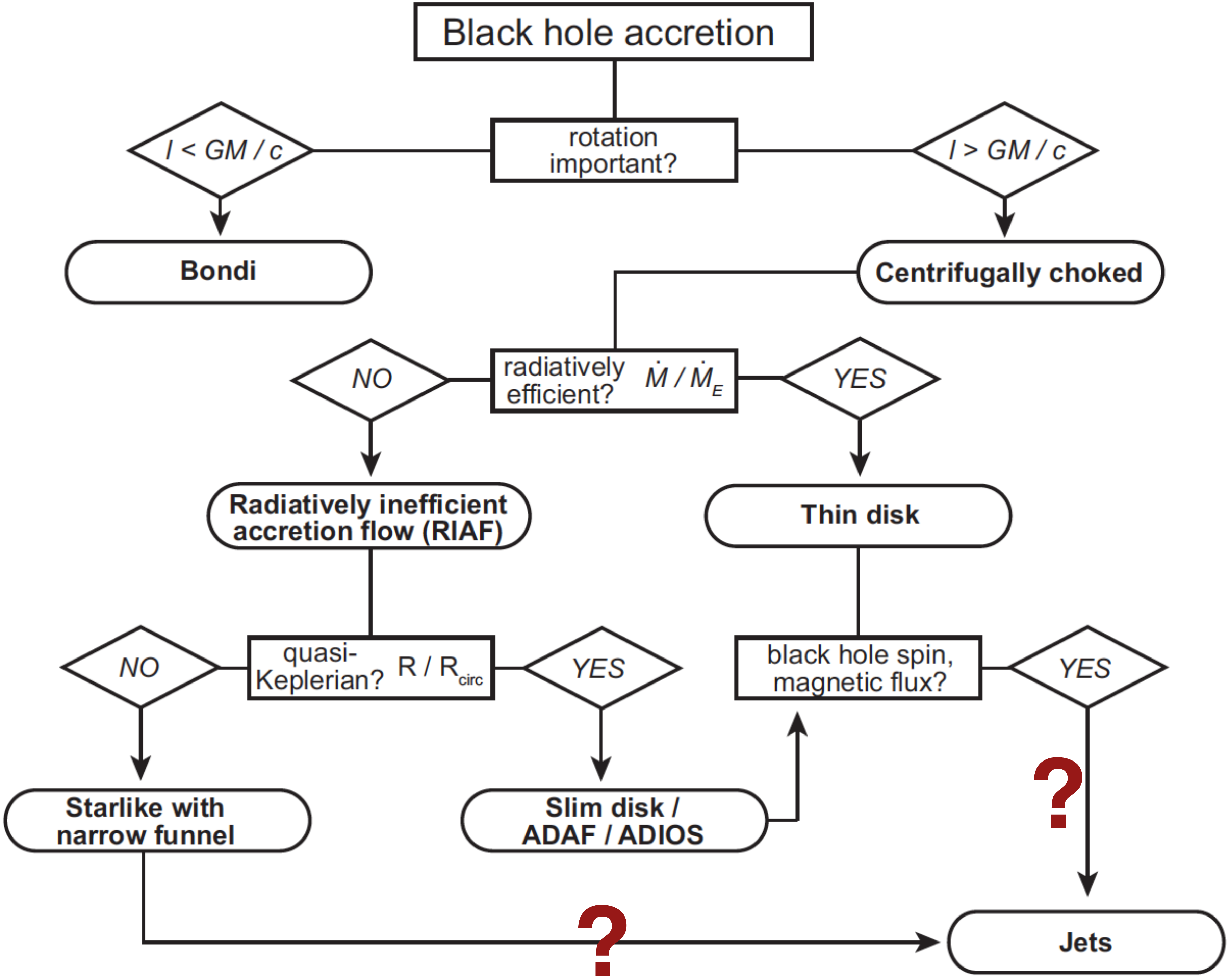
Begelman 2014



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Jets

Relativistic MHD

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Lecture 2: Fundamentals of Relativistic MHD

- Nonrelativistic hydrodynamics
- Relativistic hydrodynamics
- Nonrelativistic MHD
- Relativistic MHD
- Cleanup: gravity, conservation laws, black holes

Particle number conservation:

$$\partial_t(\sqrt{-g}\,\rho_o u^t) = -\partial_i(\sqrt{-g}\,\rho_o u^i) \qquad \partial_t\rho = -\nabla\cdot(\rho\mathbf{v})$$

Ideal MHD:

$$u_\mu F^{\mu\nu} = 0 \qquad \mathbf{E} + \mathbf{v}\times\mathbf{B}/c = 0$$

Momentum and energy conservation:

$$\partial_t\left(\sqrt{-g}\,T^t{}_\nu\right) = -\partial_i\left(\sqrt{-g}\,T^i{}_\nu\right) + \sqrt{-g}T^\kappa{}_\lambda\Gamma^\lambda{}_{\nu\kappa}$$

$$\partial_t(\rho\mathbf{v}) = -\nabla\cdot\mathbf{T} - \rho\nabla\phi$$

$$T_{\mu\nu} = (\rho_o + u + p + \frac{b^2}{4\pi})u_\mu u_\nu + (p + \frac{b^2}{8\pi})g_{\mu\nu} - \frac{b_\mu b_\nu}{4\pi}$$

$$T_{ij} = \rho v_i v_j + (p + \frac{B^2}{8\pi})\delta_{ij} - \frac{B_i B_j}{4\pi}$$

Induction equation:

$$\partial_t(\sqrt{-g}B^i) = -\partial_j(\sqrt{-g}(u^j b^i - b^j u^i)) \quad \partial_t\mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

$$= -\nabla(\mathbf{vB} - \mathbf{Bv})$$

No monopoles constraint:

$$\partial_i(\sqrt{-g}B^i) = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

Relativistic MHD

- Lecture 1: Astrophysical Motivation
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- Lecture 4: Radiative Transport and Analysis

Lecture 3: Numerical Methods

- Review of basic equations
- Kurganov-Tadmor scheme
- Variable inversion
- Reconstruction
- Constrained transport
- Testing

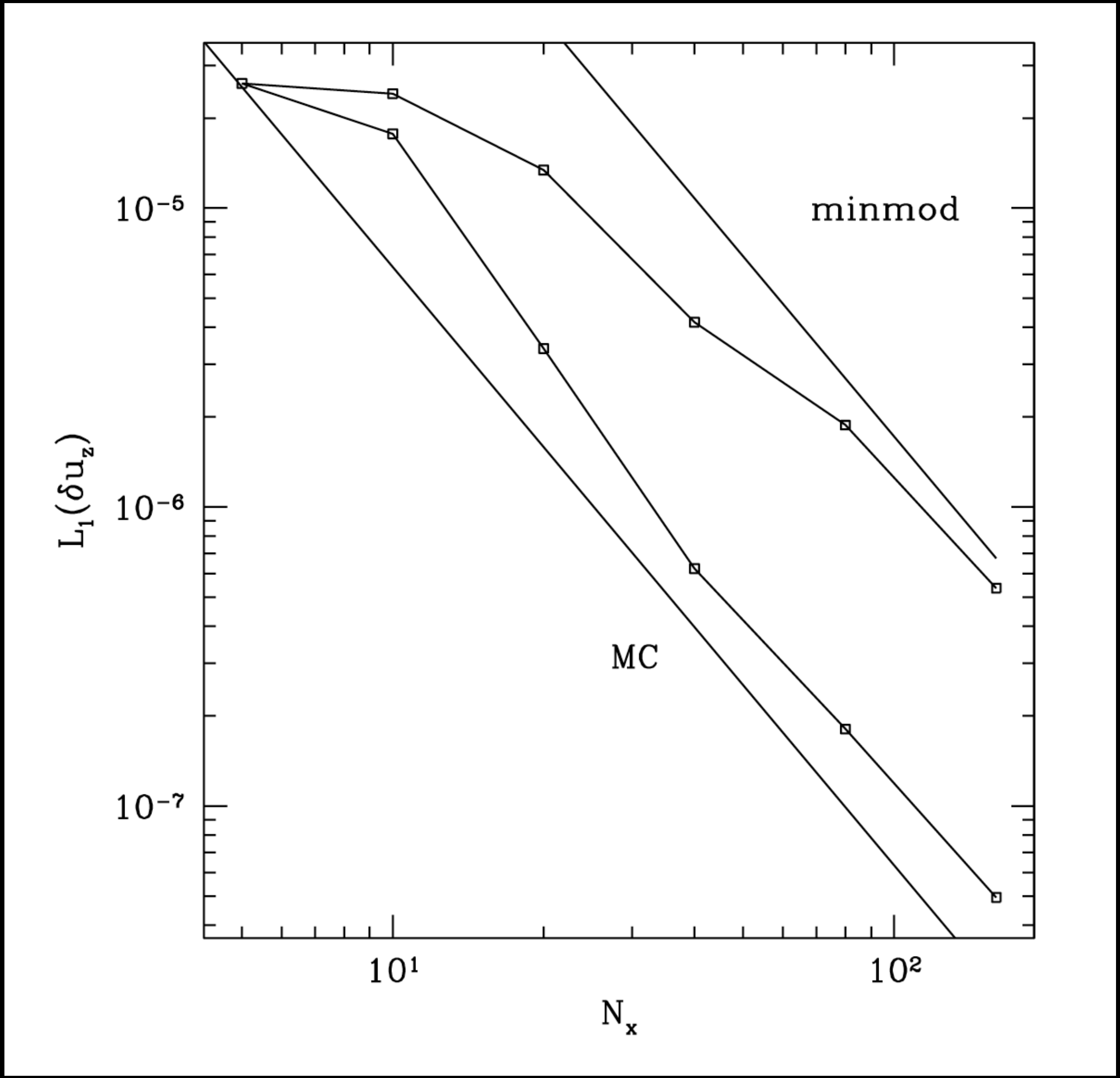
Feynman:

“the first principle is that you must not fool yourself, and you are the easiest person to fool.”

“I'm talking about a specific, extra type of integrity that is not lying, but bending over backwards to show how you're maybe wrong, [an integrity] that you ought to have when acting as a scientist. And this is our responsibility as scientists”

In computational astrophysics:

- test your code
- identify failure modes

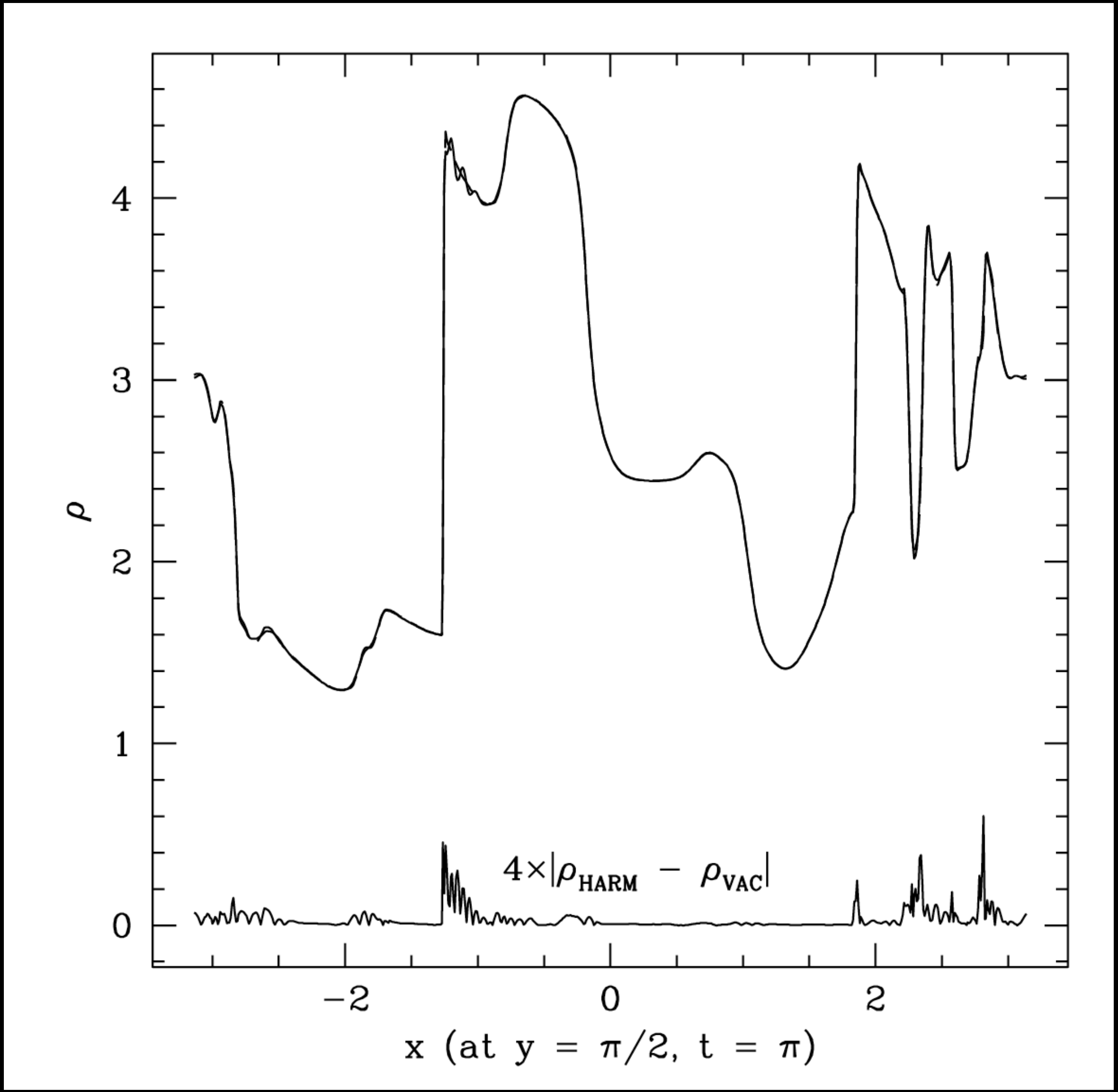


Alfven wave
test problem

Gammie+ 2003

convergence test
vs.
linear theory

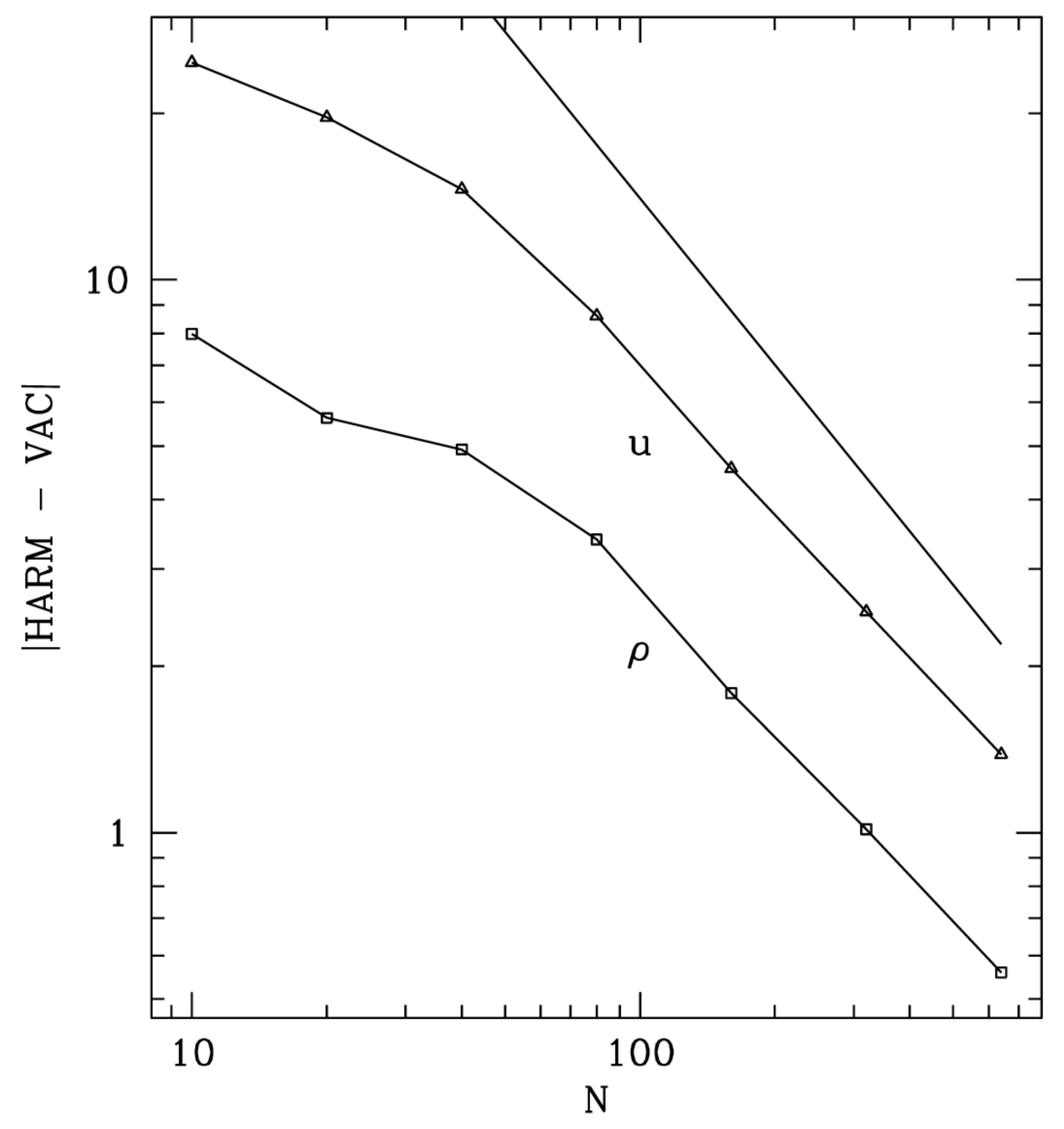
$\mathscr{L}_1(f) \equiv \int |f|\, d^2x$



Orszag-Tang
Vortex

Gammie+ 2003

nonlinear
test
vs.
VAC

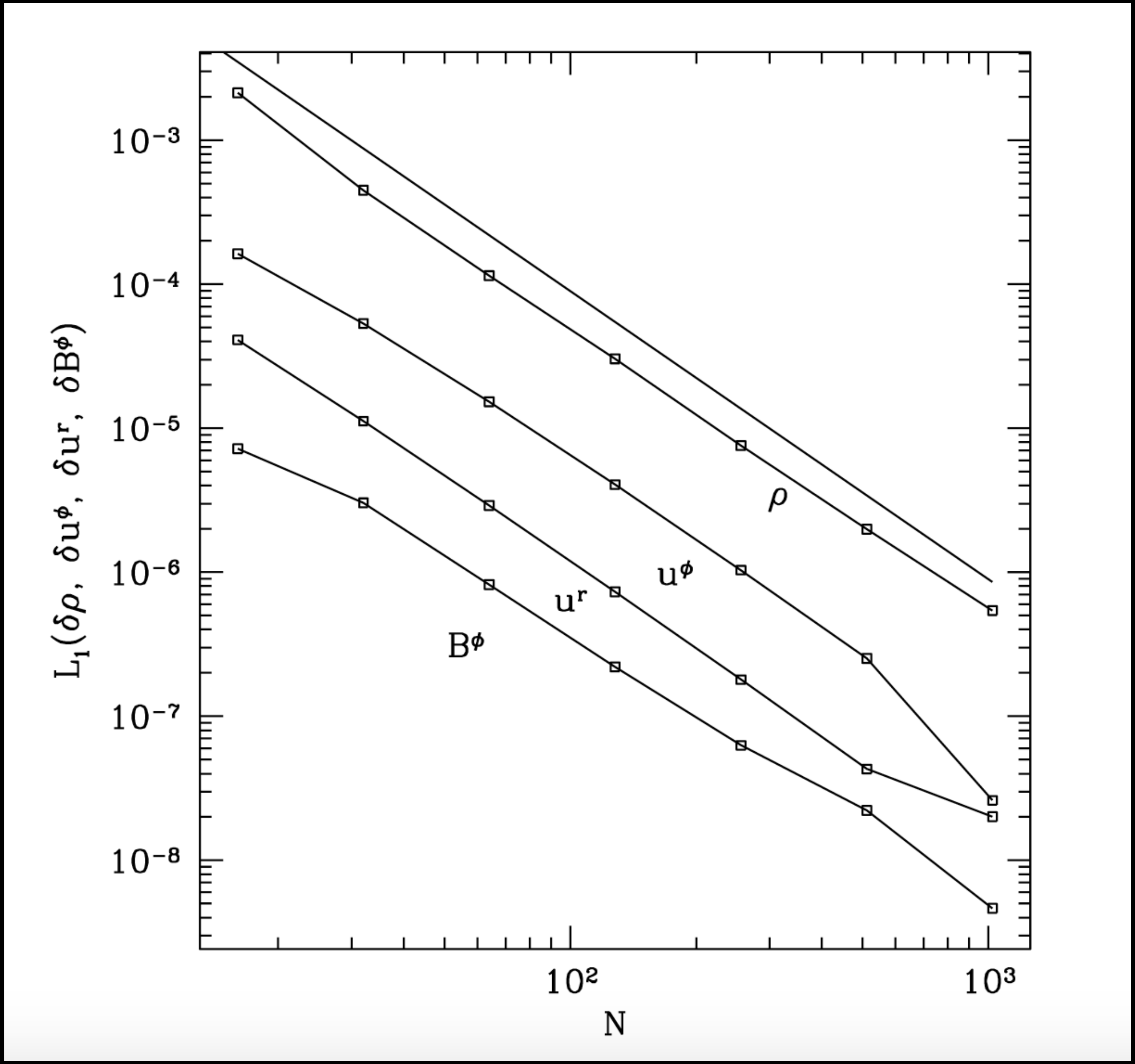


Orszag-Tang Vortex

Gammie+ 2003

convergence test vs. VAC

$\mathscr{L}_1(f) \equiv \int |f|\, d^2x$



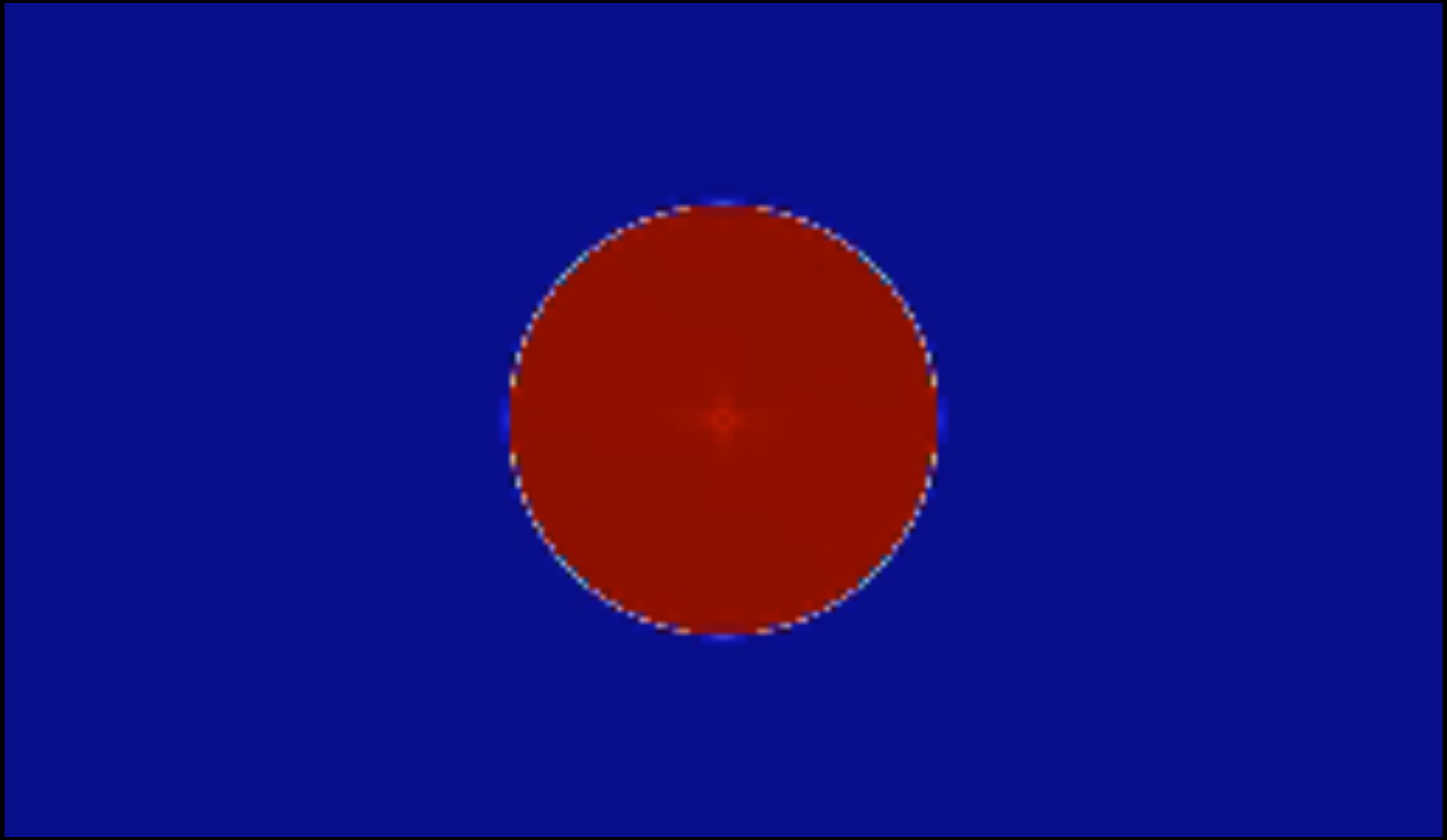
Kerr inflow
(inside-out
Parker wind)

Gammie+ 2003

convergence test
vs.
“exact” solution

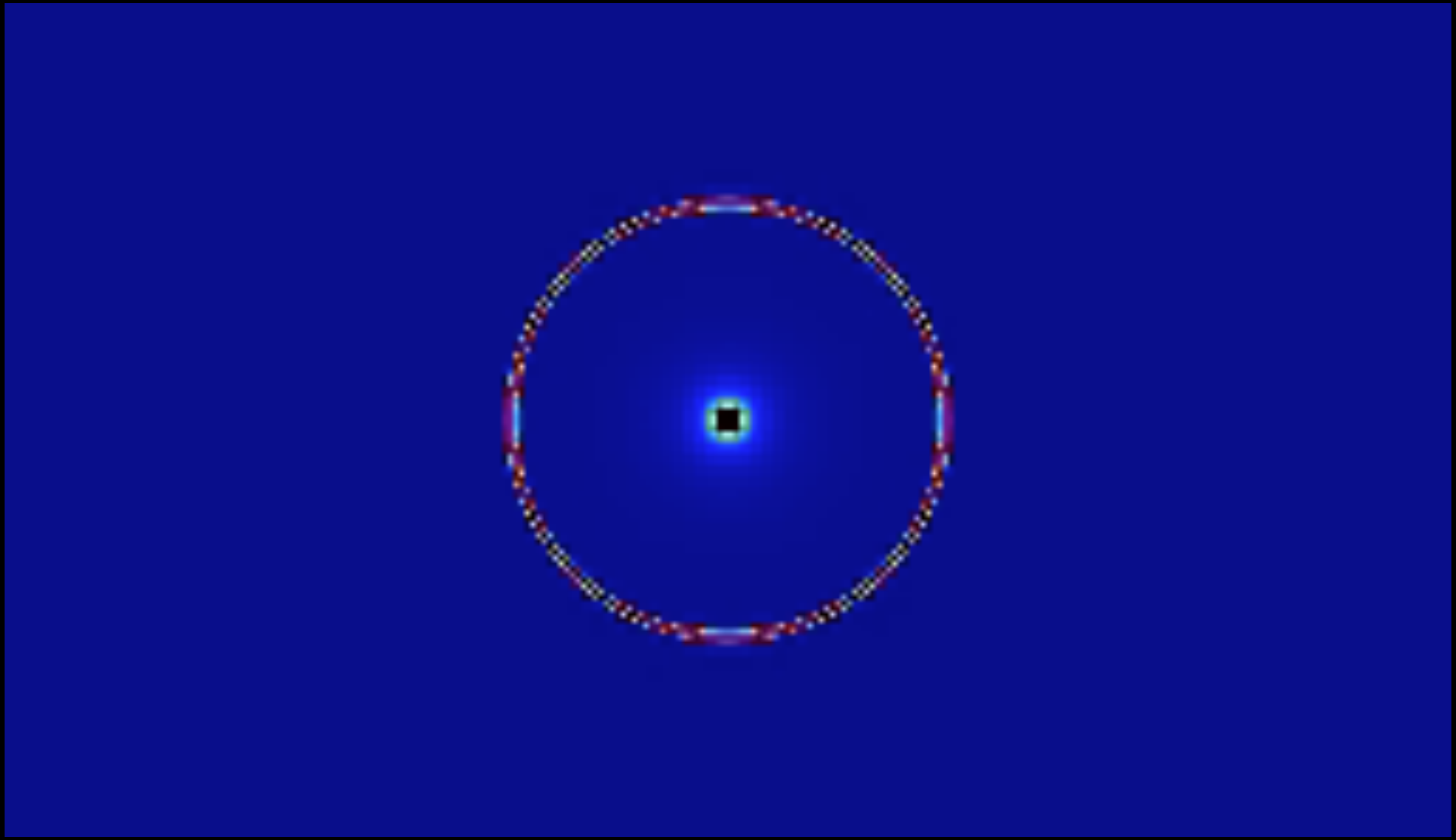
$\mathscr{L}_1(f) \equiv \int |f|\, d^2x$

Field loop advection test



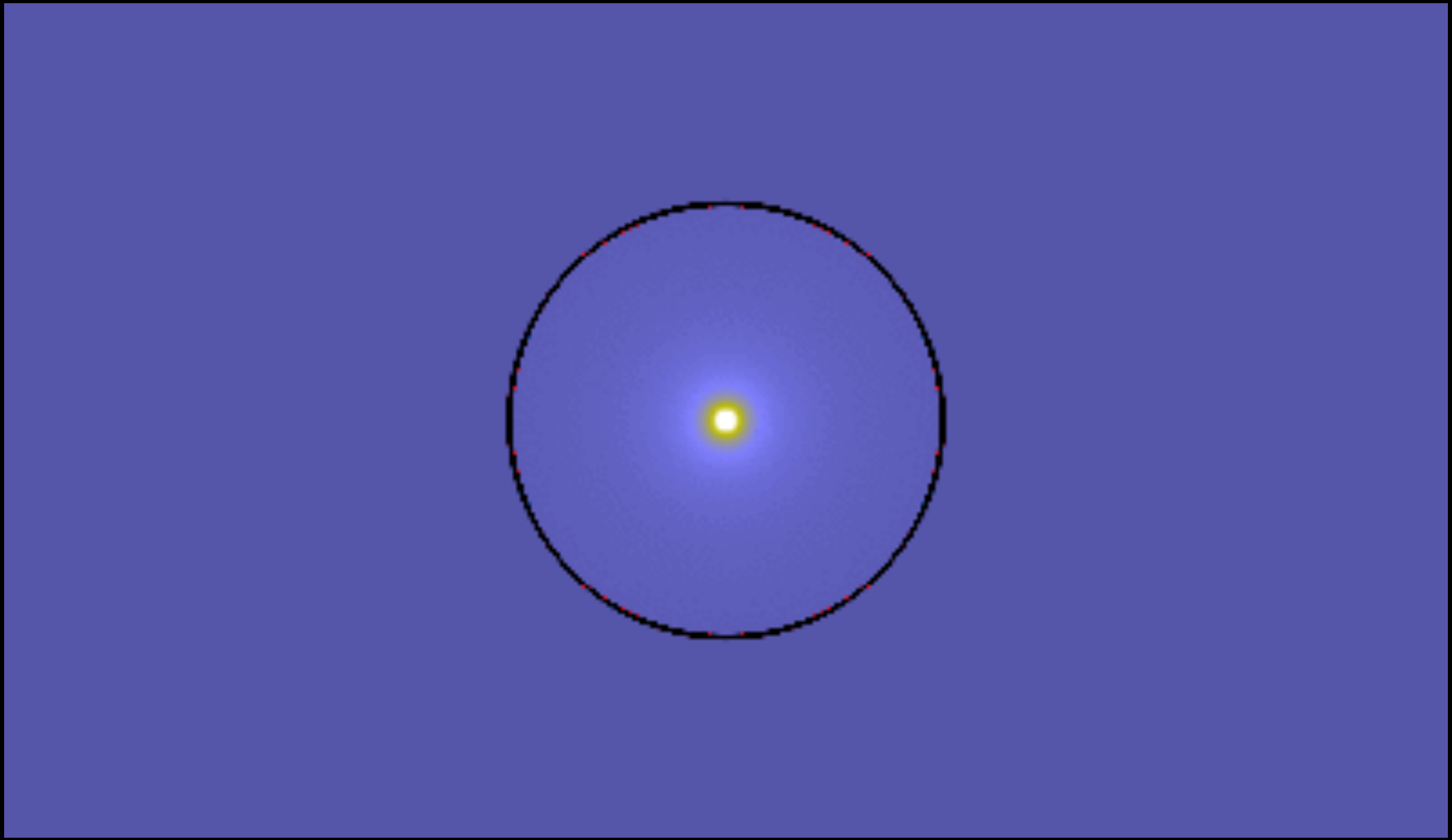
`harm` color shows b^2 $A_z \sim \mathrm{MAX}(r_0 - r, 0)$

Field loop advection test



`harm` color shows j^2 $A_z \sim \mathrm{MAX}(r_0 - r, 0)$

Field loop advection test



athena   color shows j^2 $A_z \sim \mathrm{MAX}(r_0 - r, 0)$

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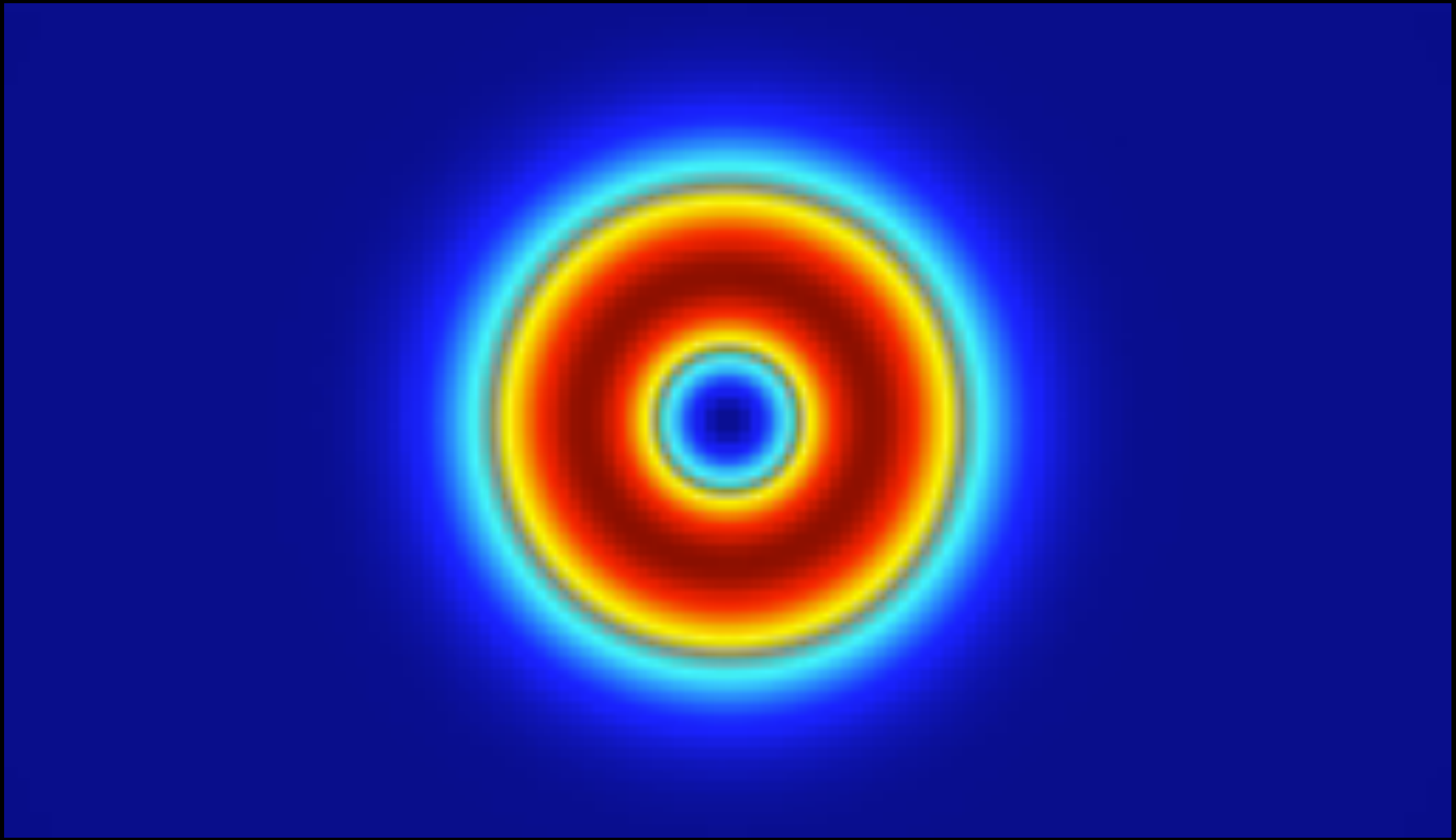
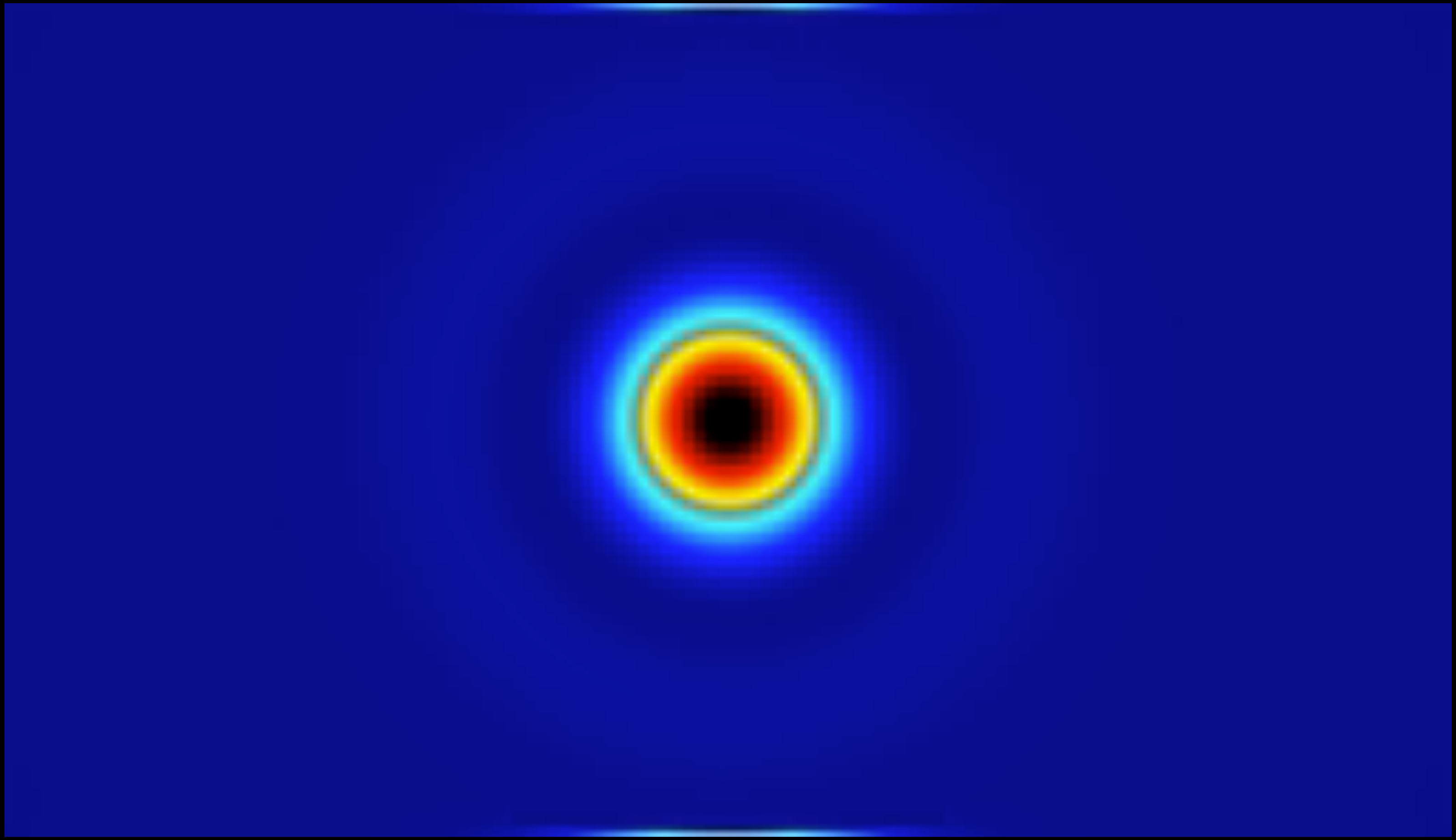


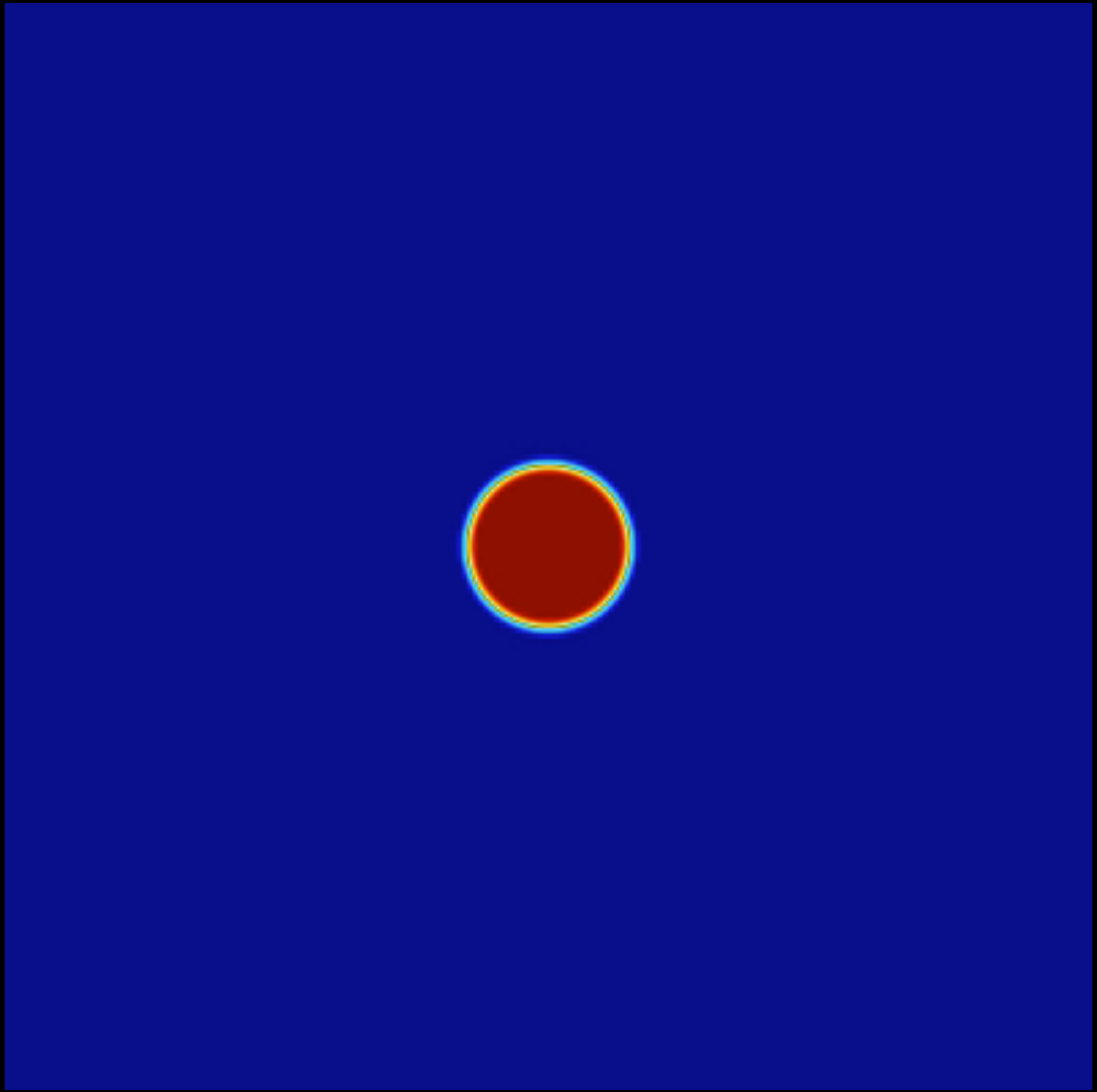
`harm` color shows b^2 $A_z \sim \exp(-r^2/w^2)$

Field loop advection test



harm color shows j^2 $A_z \sim \exp(-r^2/w^2)$

Komissarov's sadistic explosion problem



color shows log density

Further Reading

- Komissarov 1999, A Godunov-type scheme for relativistic magnetohydrodynamics, MNRAS, 303, 343-366.
- Gammie et al. 2003, HARM: A Numerical Scheme for General Relativistic Magnetohydrodynamics, ApJ, 589, pp. 444-457.
- Anile, 1990, Relativistic Fluids and Magneto-fluids, Cambridge.
- Begelman 2014, Accreting Black Holes, arXiv:1410.8132.
- Andersson & Comer 2007, Relativistic Fluid Dynamics: Physics for Many Different Scales, Living Reviews, http://www.livingreviews.org/lrr-2007-1
- Font 2008, Numerical Hydrodynamics and Magnetohydrodynamics in General Relativity, Living Reviews, http://relativity.livingreviews.org/Articles/lrr-2008-7
- Rezzolla & Zanotti 2013, Relativistic Hydrodynamics, Oxford.

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Lecture 4: Analysis and Radiative Transport

relativistic radiative transport

ibothros code:

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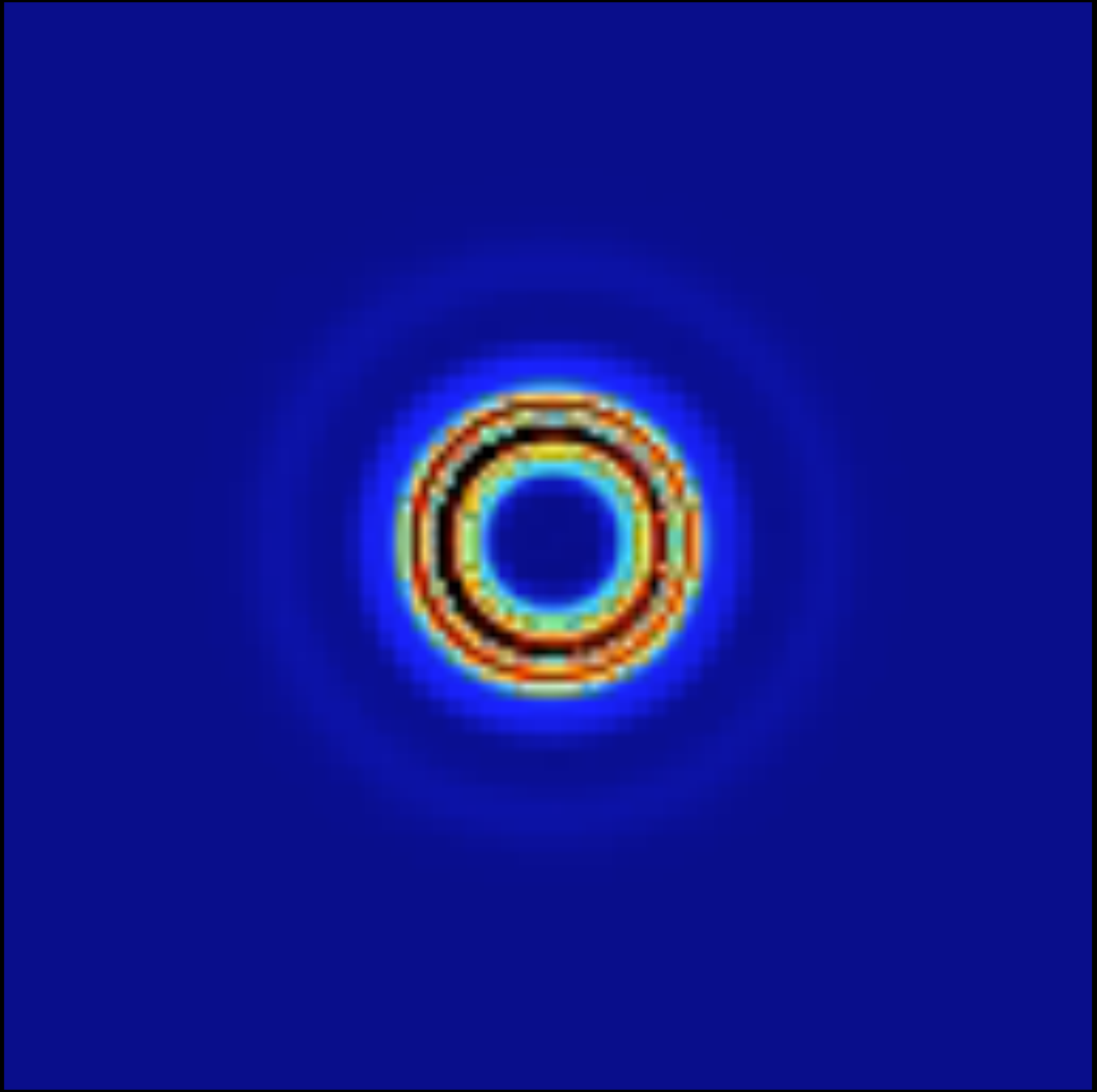
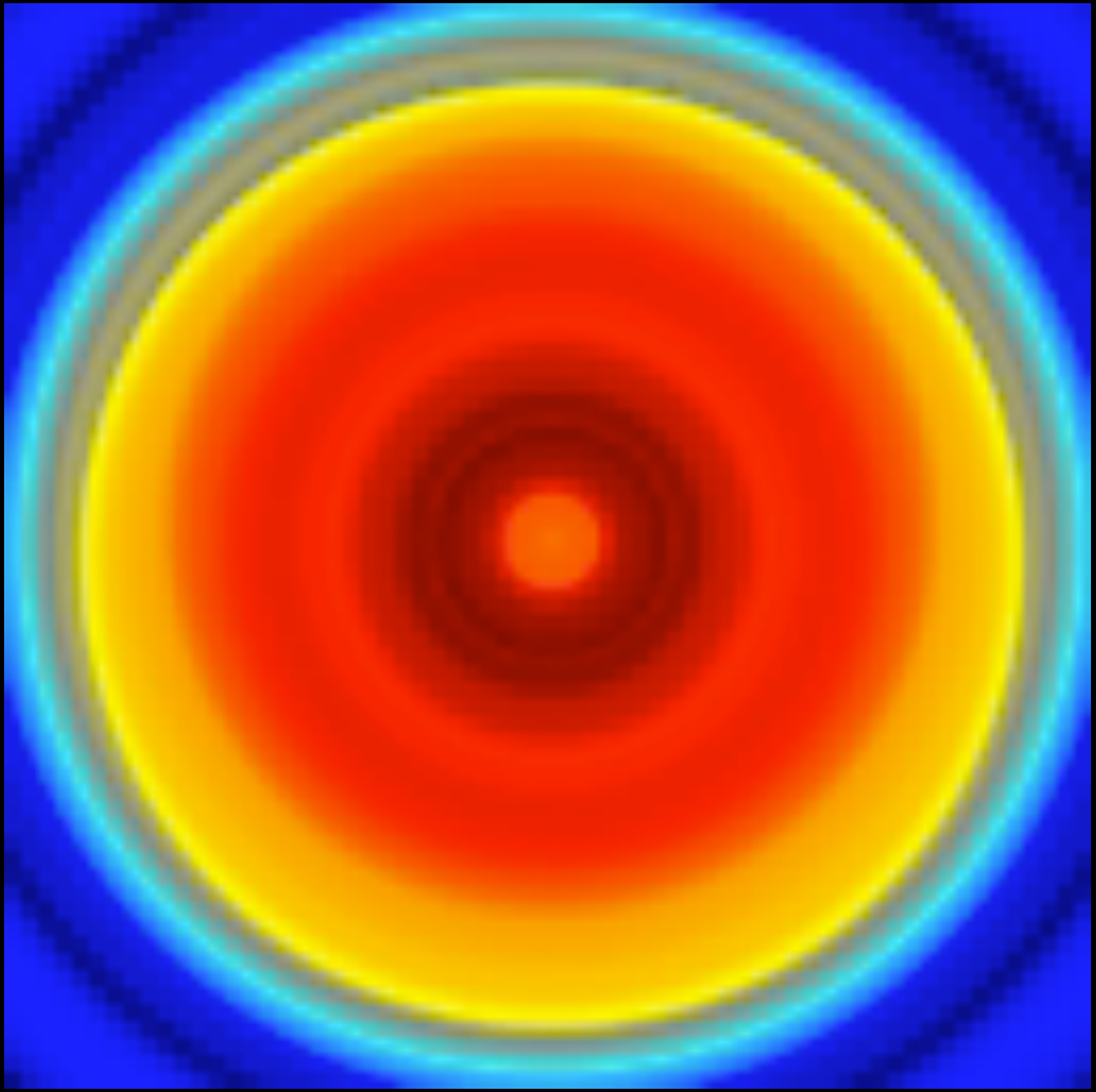
`ibothros2d` output: sweep over inclination angle



color shows I_ν at $\lambda = 1\mathrm{mm}$

`ibothros2d` output: sweep over inclination angle



color shows $\log(I_\nu)$ at λ = 1mm

Lecture 4: Radiation Transport and Analysis

- Lecture 1: Astrophysical Motivation
- Lecture 2: Basic Equations, GR Boot Camp
- Lecture 3: Numerical Methods
- Lecture 4: Radiative Transport and Analysis